A Novel Consensus-Based Optimal Control Strategy for Multi-Microgrid Systems with Battery Degradation Consideration

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Abstract—Consensus has been widely used in distributed control, where distributed individuals need to share their states with their neighbors through communication links to achieve a common goal. However, the objectives of existing consensus-based control strategies for energy systems seldom address battery degradation cost, which is an important performance indicator to assess the performance and sustainability of battery energy storage (BES) systems. In this paper, we propose a consensus-based optimal control strategy for multi-microgrid systems, aiming at multiple control objectives including minimizing battery degradation cost. Distributed consensus is used to synchronize the ratio of BES output power to BES state-of-charge (SoC) among all microgrids while each microgrid is trying to reach its individual optimality. In order to reduce the pressure of communication links and prevent excessive exposure of local information, this ratio is the only state variable shared between microgrids. Since our complex nonlinear problem might be difficult to solve by traditional methods, we design a compressive sensing-based gradient descent (CSGD) method to solve the control problem. Numerical simulation results show that our control strategy results in a 74.24% reduction in battery degradation cost on average compared to the control method without considering battery degradation. In addition, the compressive sensing method causes an 89.12% reduction in computation time cost compared to the traditional Monte Carlo (MC) method in solving the control problem.

Index Terms—consensus, distributed control, battery degradation, compressive sensing, multi-microgrid.

NOMENCLATURE

Abbreviations

BES	Battery energy storage
CSGD	Compressive sensing-based gradient descent
EI	Energy Internet
MC	Monte Carlo
MT	Microturbine

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PV	Photovoltaic
SoC	State-of-charge
WT	Wind turbine

Variables, Constants, and Parameters in Microgrids

k_i^{MT}	Control gain of MT
MG_i	The ith microgrid
$P_i^{\text{BES}}(t)$	Working power of BES
$P_i^{\text{in}}(t)$	Power transmitted to MG_i
$P_i^{\text{Load}}(t)$	Working power of load
$P_i^{\max}$	Maximum working power of BES
$P_i^{\text{MT}}(t)$	Working power of MT
$P_i^{\text{PV}}(t)$	Working power of PV
Q_i	BES battery capacity
$r_i(t)$	Ratio of BES output power to BES SoC
$SoC_i(t)$	SoC of the BES
$SoC_i^{\min}, SoC_i^{\max}$	Recommended lower and upper bounds for $SoC_i(t)$
$u_i^{\text{MT}}(t)$	Control signal applied to MT
$W_i^{\text{Load}}(t)$	Scalar Wiener process that captures the uncertainty in the change of $P_i^{\text{Load}}(t)$
$W_i^{\text{PV}}(t)$	Scalar Wiener process that captures the uncertainty in the change of $P_i^{\text{PV}}(t)$
η_i	Charging/discharging coefficient of BES
$\eta_i^{\text{in}}, \eta_i^{\text{out}}$	Charging and discharging coefficients of BES
$\rho_i^{\text{Load}}, \sigma_i^{\text{Load}}$	System coefficients related to load
ρ_i^{MT}	Constant related to MT
$\rho_i^{\text{PV}}, \sigma_i^{\text{PV}}$	System coefficients related to PV

Consensus Related

$\mathcal{A} = (a_{ij})_{n \times n}$	Adjacent matrix of graph $\mathcal{G}$
$e_i(t)$	Consensus error
$\mathcal{G}$	Undirected graph
$K = (k_{ij})_{n \times n}$	Consensus coefficient matrix
$\mathcal{N}(i)$	Neighbor set of MG_i
$u_i^r(t)$	Local state feedback function during consensus update
$\mathcal{V}, \mathcal{E}$	Vertex set and edge set of graph $\mathcal{G}$
κ_i^r	Constant in consensus update
(i, j)	Communication link (edge) between MG_i and MG_j

Control Objectives

J_{bes}	Performance indicator of BES working power overflow
J_{cons}	Performance indicator of consensus error
J_{dc}	Performance indicator of battery degradation cost
J_{soc}	Performance indicator of SoC overflow
$\epsilon_1, ..., \epsilon_4$	Significant factors of each performance indicator
$\phi_i^{\text{bes}}(t)$	Penalty function for exceeding the maximum working power of BES
$\phi_i^{\text{soc}}(t)$	Penalty function for exceeding recommended SoC bounds

CSGD Algorithm

$c_j(t)$	Sparse coefficient
$x(t), \lambda(t)$	System state and conjugate state
ξ_j	Independent identically distributed random variable sampled from the standard normal distribution
φ_j	Orthonormal basis
$\psi_j(\boldsymbol{\xi})$	Hermite polynomial

I. INTRODUCTION

Nowadays, people are paying more attention to the utilization of renewable energy due to the energy shortage crisis [1]. With a large number of renewable energy resource power generation devices, such as photovoltaics (PV) or wind turbines (WT), it becomes more and more difficult for traditional centralized energy systems to deal with the challenges of the uncertainty, nonlinearity, and intermittence of renewable energy [2]. In order to address this challenge, the concept of Energy Internet (EI) is proposed, promoting the deep integration of energy technologies and information and communication technologies. Therefore, new generation energy systems, as well as their control strategies, are more likely to work in a distributed manner [3].

In recent years, great progress has been made in the application of distributed control methods in energy systems. Compared with centralized controllers, distributed controllers cooperate to achieve a common control objective by communicating locally with their neighbors [4]. This feature plays an important role in promoting distributed control methods especially when the distribution networks of different energy forms are coupled with each other. In [5], a double-Newton descent algorithm is designed to achieve fully distributed multi-energy management in the sense that global energy trading prices are optimized through the exchange of local decision information within the neighborhood only. Considering that the restrictions of real-world communication links might impact the control effect, the authors of [6] propose a distributed control method, which improves the robustness of active and reactive heat and electric power sharing against transmission latency and bandwidth limit. The authors of [7] carry out a noniterative decoupled scheduling method for the combined heat and electric power system, and then in [8], they propose an equivalent model-based noniterative operation

strategy to improve the efficiency of the coordination in a similar way.

Consensus is a rule that tries to agree on a common value by sharing local state information in a distributed system [9] and is a fundamental problem in the study of distributed control [10]. In consensus-based control, distributed individuals only need to updates the consensus variable, i.e., the state variable to synchronize via consensus, according to the shared information received from its neighbors [11]. Consensus-based control has attracted extensive attention due to its fast convergence speed, limited information exchange, and strong operation collaboration [12, 13]. The most prominent advantage of consensus-based control is that it helps to maintain the stability of the system in case of disturbance or failure [14, 15].

In energy systems, especially in multi-microgrid systems, consensus-based control is mainly adopted as a secondary control scheme for economic dispatch [16], reactive and active power sharing [17], and battery energy storage (BES) system management [18]. The collaborative feature of consensus-based control is also proficient in maintaining system stability when the plug-and-play feature is enabled [19]. By selecting different variables to reach consensus and state information to share between microgrids, consensus-based control can achieve many different control objectives such as social welfare maximization, frequency/voltage regulation, and SoC balance. Table I provides a detailed summary of the related works on consensus-based control in energy systems.

In addition, when performing control on BES devices, battery degradation is an important performance indicator to consider in order to prevent overcontrol and extend battery lifetime [29, 30]. Recently, battery degradation cost has become an economic factor that is frequently included in many related problems [29, 31, 32, 33]. In spite of this, there is no consensus on how battery degradation cost is calculated. Some works model degradation cost as a function of the depth of discharge of the battery [34, 35], while others might believe that the charging and discharging rates of the BES can also affect the degradation degree in the battery [36, 37].

A. Motivations

The consensus algorithms adopted by most existing works require that each distributed individual updates the consensus variable by a linear combination of that of its neighbors. These algorithms are distinguished by their consensus coefficients, i.e., the choice of the coefficients for the linear combination during state update. Local adjacency [10] is the most used consensus algorithm. The problem of adopting the consensus coefficients of well-known algorithms is nevertheless that once the update rules are known, the exact values of some local states could be computable by others [38]. In order to prevent unexpected information exposure, it would be reasonable to find a new set of consensus coefficients for each control problem.

Privacy is a practical concern in distributed control when microgrids are owned and operated by different agents in the absence of mutual trust. However, related works tend to share too many local state variables during the process of

TABLE I
RELATED WORKS ON CONSENSUS-BASED CONTROL IN ENERGY SYSTEMS

Scenario	Control Objective	Consensus Variable	Shared State Information	Reference
Economic dispatch	Social welfare maximization	Incremental cost	Estimated power mismatch	[16]
	Minimizing generation cost	Incremental cost	Power mismatch	[20]
	Social welfare maximization	Cost of consumed energy, cost of produced energy	Cost of consumed energy, cost of produced energy	[21]
Reactive and active power sharing	Voltage regulation	Critical bus voltage, cumulative time after triggering on load tap charger	Critical bus voltage, cumulative time interval after triggering on load tap charger, measured reactive power, predicted reactive power	[22]
	Voltage regulation	Reative power, voltage amplitude	Reative power, voltage amplitude	[23]
	Frequency regulation	Reative power, active power	Reative power, active power, control parameters of droop controllers	[24]
	Energy consumption reduction	Heat/electric power sharing error	Proportional change of heat/electric power and its first deviation, auxiliary control signal, heat/electric power sharing error	[25]
BES management	SoC balancing	Relative SoC variation	Estimated average of lumped consensus parameter	[26]
	power control	SoC, output power	SoC, output power, voltage, current	[27]
	Voltage regulation	Estimation of average SoC	Utilization ratio	[19]
	PV voltage control	Average bus voltage	Ratio of BES output power to SoC, estimated average voltage	[28]

consensus (e.g., [22, 24, 25]). According to the authors of [16], any local state variable can be seen as private information. Similarly, [39] recommends that nothing about the distributed individual should be revealed, and state variables and their initial values need to be protected. Since local control data might contain sensitive information, such as local energy utilization of distributed users, oversharing of local information could not only increase the risk of privacy exposure, but also impose great burden on communication links. This problem can be effectively mitigated by sharing as few state variables as possible. Intermediate variables, on the contrary, are considered noncritical and are safe for sharing among distributed microgrids [16]. In addition, adding a noise to each variable before it is shared is another way to enhance privacy [38].

There have been many works synchronizing each BES SoC by consensus because batteries will have higher efficiency and longer life if their SoCs are kept in a certain range [40, 41]. However, simple SoC synchronization could instead cause overcharging/overdischarging and shorten the lifetime of batteries when considering the impact of uncertain line resistance [27]. Moreover, different initial SoC conditions of batteries might produce circulating currents, and the performance degradation caused by the circulating currents could be alleviated by agreeing on a common value of this ratio [28]. As a result, taking this ratio as the consensus variable will lead to nonlinearity in the control problem, which may be difficult to solve.

In light of this, works such as [27] and [28] have proposed different strategies to simplify the nonlinear control problem mentioned above. In [28], the SoCs of all BESs are assumed to be constant so that the complicated nonlinear control problem can be approximated by a linear problem, which oversimplifies the problem and does not conform to the reality. On the contrary, the control method in [27] independently updates both output power and SoC of a BES by their individual consensus update equations. The problem is that since SoC is usually regarded as a function of the BES output power, this method may not work as it has been assumed. In fact, there are some works that do not try to simplify the nonlinearity of their problems. For example, the state update equation for the consensus-based control for reactive power sharing in [42] has a nonlinear form. Although its equilibrium and convergence can be theoretically proved, the form of its consensus update equation is so specialized that it can hardly be extended to other problems. It is recommended that the complexity of the problem should be maintained so as to make the control strategy conform to the practical scenarios.

In microgrids, BES devices are mainly used to absorb voltage and frequency fluctuations [43]. Inappropriate operation of the BES that ignores battery degradation could shorten the battery lifespan and even damage the system stability [44]. Unfortunately, existing works on consensus-based control seldom consider battery degradation cost. Although different models have been applied to evaluate battery degradation, they are usually nonlinear functions of the factors considered [45], which can greatly increase the difficulty of finding the optimal control strategy. It would be more appropriate if the control objective addresses the nonlinearity of minimizing of battery degradation cost.

The dynamic programming method and the Monte Carlo (MC) method are widely used in solving engineering control problems [46, 47]. However, when solving high dimensional problems, the dynamic programming method suffers

the curse of dimensionality, i.e., the significant computational workload makes the problem almost impossible to solve as the dimension of the problem increases. Moreover, the MC method usually has low efficiency in order to meet a practical computation accuracy. The compressive sensing method, on the other hand, emerged from the study of sparse signal recovery [48]. It takes advantage of the sparsity of signals to restore the original signals from fewer sampling points. Compressive sensing has been applied to solve various high-dimensional problems, such as image reconstruction [49], network topology identification [50], and routing for sensor networks [51]. Therefore, the compressive sensing method can be used to overcome the difficulty of solving high-dimensional control problems, which is hardly seen in existing works.

B. Contributions

The contributions of this paper are summarized as follows:

1) This paper proposes a consensus-based optimal control strategy for the distributed control of a regional multi-microgrid network consisting of loads, PVs, and BESs. The optimization objective includes the minimization of battery degradation cost caused by BES control, which is seldom considered by existing consensus-based control methods. Distributed consensus is used to synchronize the ratio of BES output power to BES SoC among microgrids. Compared with most works summarized in Table I, our control strategy chooses this ratio as the only shared information between microgrids, which can not only reduce the pressure on communication links, but also avoid excessive exposure of local information. Our control strategy reduces battery degradation cost by 74.24% compared to the control method without considering battery degradation.

2) Our method of solving the control problem is deep in theory as it integrates the knowledge and means of mathematics, control theory, and computer science. In more detail, we first model the multi-microgrid system by stochastic differential equations and formulate the control problem as an optimization problem. Then, we solve the optimization problem for consensus coefficients instead of adopting the most popular consensus coefficients in case of unexpected information exposure. Instead of simplifying the nonlinear problem for solvability like [27, 28] that might cause discrepancy from actual situations, we use a compressive sensing method to solve the control problem while the nonlinearity of the problem is preserved. Note that although compressive sensing has become a popular strategy to deal with high-dimensional problems, it has barely been applied in solving practical control problems. This paper designs a compressive sensing-based gradient descent (CSGD) method to solve our nonlinear control problem. The computation efficiency is improved by 89.12% compared with the traditional MC method, and the number of sample points required is greatly reduced.

The remainder of this paper is organized as follows: Section II models the dynamics of the multi-microgrid system we
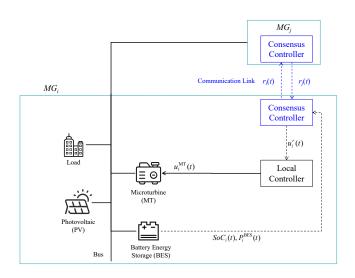

Fig. 1. A typical microgrid.

study and explains in detail the proposed consensus-based control strategy; Section III describes the control objectives and formulates the control strategy into an optimization problem; Section IV proposes CSGD that solves the optimization problem based on compressive sensing; Section V evaluates the performance of our control strategy by simulation; Section VI concludes this paper.

II. SYSTEM DESCRIPTION

In this paper, we consider a system composed by n microgrids, each consisting of a load, a PV, a microturbine (MT), and a BES device (as shown by Fig. 1). In addition to physical links, communication links can also be established between microgrids to enable state information sharing. Denote the ith microgrid by MG_i.

In this section, we describe the dynamics of the multi-microgrid system by stochastic differential equations and provide details of the novel consensus-based control strategy based on this system.

A. Dynamics of a Single Microgrid

Now we provide the description of the system dynamics of MG_i.

1) Dynamics of Load and Photovoltaic: Note that the power of the load and the PV could be affected by environmental disturbances (e.g., unexpected access of a high-power device or sudden change of light intensity). This kind of randomness and uncertainty can be modeled by stochastic differential equations.

Specifically, define a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, with Ω being the sample space, $\mathcal{F}$ being the σ-algebra of the subsets of Ω, and $\mathbb{P}$ being a probability measure. We can define two independent scalar Wiener processes $W_i^{\text{Load}}(t)$ and $W_i^{\text{PV}}(t)$ in the probability space, which describe the short-term power deviation in load and PV. Then the dynamics of the load and the PV of MG_i can be modeled as follows [52, 53, 54]:

$$\mathrm{d}P_i^{\text{Load}}(t) = -\rho_i^{\text{Load}} P_i^{\text{Load}}(t)\mathrm{d}t + \sigma_i^{\text{Load}}\mathrm{d}W_i^{\text{Load}}(t), \quad (1)$$

$$dP_i^{\mathrm{PV}}(t) = -\rho_i^{\mathrm{PV}} P_i^{\mathrm{PV}}(t)dt + \sigma_i^{\mathrm{PV}}dW_i^{\mathrm{PV}}(t), \qquad (2)$$

where $P_i^{\mathrm{Load}}(t)$, $P_i^{\mathrm{PV}}(t)$ are the working power of load and PV, and ρ_i^{Load}, ρ_i^{PV}, σ_i^{Load}, σ_i^{PV} are system coefficients.

The dynamics of stochastic power systems has been studied since decades ago [55, 56]. When the energy system encounters uncertainties (such as the unpredictable changes of solar radiation, wind speed, or load power), some of the system parameters cannot be accurately measured by traditional methods due to modeling errors [53, 57]. For such dynamic systems, the stochastic deviations cannot be simply described by the ordinary differential equations. It is notable that stochastic differential equations have been used to describe random power deviations; see, e.g., [52, 53, 58].

2) Dynamics of Battery Energy Storage: SoC is an important indicator that evaluates the remaining capacity of a BES device. The SoC of the ith BES is defined as [59]:

$$dSoC_i(t) = -\frac{\eta_i}{3600Q_i} P_i^{\mathrm{BES}}(t)dt, \qquad (3)$$

where Q_i is the battery capacity, $P_i^{\mathrm{BES}}(t)$ is the output power of the BES, and η_i is the charging/discharging coefficieint defined by:

$$\eta_i = \begin{cases} \eta_i^{\mathrm{in}} & P_i^{\mathrm{BES}}(t) < 0 \\ 1/\eta_i^{\mathrm{out}} & P_i^{\mathrm{BES}}(t) \geq 0 \end{cases}$$

The parameters $\eta_i^{\mathrm{in}}, \eta_i^{\mathrm{out}}$ are related to the charging and discharging efficiency of the BES. Note that the factor 3600 on the denominator of (3) arises due to unit conversion when $P_i^{\mathrm{BES}}(t)$ is in kW, t is in s, and Q_i is in kW·h.

The output power of the BES should be limited within a proper range:

$$0 \leq |P_i^{\mathrm{BES}}(t)| \leq P_i^{\mathrm{max}}, \qquad (4)$$

where P_i^{max} is the maximum working power of the BES. In addition, the constraints for the BES SoC is given by:

$$0 < SoC_i(t) < 1. \qquad (5)$$

In order to prevent overcharging and overdischarging, we recommend that $SoC_i(t)$ is limited within the following bounds:

$$SoC_i^{\mathrm{min}} \leq SoC_i(t) \leq SoC_i^{\mathrm{max}}, \qquad (6)$$

where SoC_i^{min} and SoC_i^{max} are the recommended lower and upper bounds for $SoC_i(t)$, in respective. Note that the bounds defined by (6) are soft in the sense that the value of $SoC_i(t)$ might run out of the recommended range during the actual operation, but it will be subjected to penalties (as will be described in Section III-A3). On the contrary, $SoC_i(t)$ needs to strictly follow the requirement given by (5).

3) Dynamics of Microturbine: The uncertainty of renewable energy lead to the imbalance of power supply and demand in microgrids, thus seriously affecting the normal operation of the whole system. Therefore, we add an MT that consumes traditional fossil energy in each microgrid as a controllable power generation device. By controlling the power generation of the MT, the shortage of PV power generation can be made up.

Based on [54], the dynamics of the MT can be modeled as:

$$dP_i^{\mathrm{MT}}(t) = -\rho_i^{\mathrm{MT}} \left[P_i^{\mathrm{MT}}(t) - k_i^{\mathrm{MT}} u_i^{\mathrm{MT}}(t) \right] dt, \qquad (7)$$

where $P_i^{\mathrm{MT}}(t)$ is the output power of the MT, $u_i^{\mathrm{MT}}(t) \in [0, 1]$ is the control input signal applied to the MT, ρ_i^{MT} is a constant, and k_i^{MT} is the control gain.

4) Power Balance: When the power supply does not match the power demand within a microgrid, power transmission will take place between connected microgrids. If the power supply exceeds the demand in MG_i, the oversupply can be used to power other microgrids. Otherwise, the shortage in supply is mitigated by the power transmission from other microgrids. Suppose $P_i^{\mathrm{in}}(t)$ is the power transmitted to MG_i. When $P_i^{\mathrm{in}}(t) > 0$, energy flows from other microgrids into MG_i, and it is the other way if $P_i^{\mathrm{in}}(t) < 0$. As a result, the power in MG_i should reach a balance:

$$P_i^{\mathrm{PV}}(t) + P_i^{\mathrm{MT}}(t) + P_i^{\mathrm{BES}}(t) + P_i^{\mathrm{in}}(t) = P_i^{\mathrm{Load}}(t). \qquad (8)$$

B. Consensus-Based Control

We now describe a novel consensus-based control strategy for the management of the BES system.

The communication network of the system can be modeled as an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where vertex set $\mathcal{V} = \{1, 2, \ldots, n\}$ corresponds to the collection of microgrids $\{MG_1, MG_2, \ldots, MG_n\}$. Edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ contains all the connection between microgrids. In more detail, edge $(i, j) \in \mathcal{E}$ if and only if there is a communication connection between MG_i and MG_j. Let $\mathcal{N}(i) \subseteq \mathcal{V}$ be the neighbor set of MG_i:

$$\mathcal{N}(i) = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}.$$

It contains all the microgrids connected to MG_i.

Then, we can define the adjacent matrix $\mathcal{A} = (a_{ij})_{n \times n}$ as follows:

$$a_{ij} = \begin{cases} 1 & \text{if } (i, j) \in \mathcal{E}, \\ 0 & \text{otherwise}. \end{cases}$$

Usually, $a_{ii} = 0$ for all $i \in V$, meaning that the microgrid does not need a communication link to itself. In addition, the undirected graph require that edge (i, j) and (j, i) stand for the same communication link between MG_i and MG_j. As a result, $a_{ij} = a_{ji}$ for all $i, j \in \mathcal{V}$. This adjacent matrix is used to algebraically depict the topological structure of the multi-microgrid network and plays an important role in consensus.

In distributed consensus, microgrids will reach an agreement on some common state variable via communication links. Consensus is achieved when the state variables of all microgrids are equal [16]. This consensus value can be seen as an equilibrium point where microgrids reach a balance between local control and global collaboration [60, 61].

SoC synchronization is classical application of consensus in energy Internet with BES systems [40, 41]. However, as we have mentioned in Section I, simple SoC synchronization could cause degradation in system performance by causing unexpected overcharging/overdischarging and circulating currents (readers can refer to [27, 28] for detailed arguments). Therefore, the goal of the consensus in our control strategy is to synchronize the ratio of BES output power to BES SoC like [27, 28].

Formally, the ratio of BES output power $P_i^{\text{BES}}(t)$ to BES SoC $SoC_i(t)$, denoted by $r_i(t)$, is defined as follows:

$$r_i(t) = \frac{P_i^{\text{BES}}(t)}{SoC_i(t)}, \qquad (9)$$

which captures the relative variation rate of $SoC_i(t)$. A consensus algorithm requires that MG_i updates consensus variable $r_i(t)$ based on the linear combination of the corresponding states of its neighbors as the following (continuous form) consensus update equation [62]:

$$\dot{r}_i(t) = \kappa_i^r u_i^r(t), \qquad (10)$$

where κ_i^r is a constant, and $u_i^r(\cdot)$ is a local state feedback function defined as:

$$u_i^r(t) = \sum_{i=1}^{n} k_{ij}(r_j(t) - r_i(t)), \qquad (11)$$

where $\{k_{ij}\}_{i,j=1}^{n}$ is a set of consensus coefficients to be determined. Since state information is only shared locally between adjacent microgrids, we have $k_{ij} = 0$ if $j \notin \mathcal{N}(i)$. Moreover, the ratio $r_i(t)$ is the only state variable to be shared with neighbor microgrids through communication links.

We say that consensus is achieved if all $r_i(t)$ eventually become stable at a fixed value (or equilibrium point) r^* [20]:

$$\lim_{t \to \infty} r_1(t) = \lim_{t \to \infty} r_2(t) = \ldots = \lim_{t \to \infty} r_n(t) = r^*. \qquad (12)$$

In other words, the goal of consensus is to synchronize the relative change speed in the SoCs of all BESs. We can define the consensus error $e_i(t)$ of MG_i as follows [63]:

$$e_i(t) = \sum_{i=1}^{n} a_{ij}(r_j(t) - r_i(t)). \qquad (13)$$

It captures the difference of r_i of MG_i with that of its neighbors.

In the presence of the Wiener processes $W_i^{\text{Load}}(t)$ and $W_i^{\text{PV}}(t)$, the strict consensus requirement described by (12) may not be achievable. In this case, the consensus requirement is relaxed as [64]:

$$\lim_{t \to \infty} \mathbb{E}[|r_i(t) - r^*|] = 0, \quad i = 1, 2, \ldots, n, \qquad (14)$$

where $\mathbb{E}[\cdot]$ stands for mathematical expectation. In other words, we say that consensus is reached as long as all $r_i(t)$ have the same expectation r^* even if they are not stabilized.

By combining (3) (9) and (10), we have:

$$dP_i^{\text{BES}}(t) = \left[SoC_i(t)u_i^r(t) - \frac{\eta_i \left(P_i^{\text{BES}}(t) \right)^2}{3600Q_i SoC(t)} \right] dt. \qquad (15)$$

In other words, the update of the ratio $r_i(t)$ in (11) is achieved by changing the BES output power $P_i^{\text{BES}}(t)$. However, the BES is not a controllable device. Its power can only be changed when the multi-microgrid network is trying to keep the power balance according to (8). Therefore, we can force the change in the BES power by controlling the MT to break the power balance.

C. Dynamics of the Multi-Microgrid System

Now we describe the comprehensive system model of the multi-microgrid EI system.

Let

$$x_i(t) = \left[P_i^{\text{Load}}(t), P_i^{\text{PV}}(t), SoC_i(t), r_i(t), P_i^{\text{MT}}(t) \right]^{\text{T}},$$

$$u_i(t) = \left[u_i^r(t), u_i^{\text{MT}}(t) \right]^{\text{T}}, \quad W_i(t) = \left[W_i^{\text{Load}}(t), W_i^{\text{PV}}(t) \right]^{\text{T}},$$

where the superscript "T" means matrix (vector) transpose. Combining (1), (2), (3), (10), (7), the system dynamics of MG_i can be rewritten as:

$$dx_i(t) = [A_i x_i(t) + f_i(x_i(t)) + B_i u_i(t)] \, dt + S_i dW_i(t), \qquad (16)$$

where

$$A_i = \begin{bmatrix} -\rho_i^{\text{Load}} & 0 & 0 & 0 & 0 \\ 0 & -\rho_i^{\text{PV}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\rho_i^{\text{MT}} \end{bmatrix},$$

$$B_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \kappa_i^r & 0 \\ 0 & \rho_i^{\text{MT}} k_i^{\text{MT}} \end{bmatrix}, \quad S_i = \begin{bmatrix} \sigma_i^{\text{Load}} & 0 \\ 0 & \sigma_i^{\text{PV}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

are constant matrices, and

$$f_i(x_i(t)) = \left[0, 0, 0, -\frac{\eta_i}{3600Q_i} r_i(t)SoC_i(t), 0 \right]^{\text{T}}$$

is a nonlinear function of $x_i(t)$.

Further, let

$$x(t) = \left[x_1(t)^{\text{T}}, x_2(t)^{\text{T}}, \ldots, x_n(t)^{\text{T}} \right]^{\text{T}},$$

$$u(t) = \left[u_1(t)^{\text{T}}, u_2(t)^{\text{T}}, \ldots, u_n(t)^{\text{T}} \right]^{\text{T}},$$

$$W(t) = \left[W_1(t)^{\text{T}}, W_2(t)^{\text{T}}, \ldots, W_n(t)^{\text{T}} \right]^{\text{T}}.$$

Then the dynamics of the multi-microgrid EI system can be rewritten as:

$$dx(t) = [Ax(t) + f(x(t)) + Bu(t)] \, dt + Sx(t)dW(t), \qquad (17)$$

where

$$A = \text{diag}[A_1, A_2, \ldots, A_n], \quad B = \text{diag}[B_1, B_2, \ldots, B_n],$$
$$S = \text{diag}[S_1, S_2, \ldots, S_n]$$

are constant quasi-diagonal matrices with the diagonal blocks being the matrices in $\text{diag}[\ldots]$, and

$$f(x(t)) = \left[f_1(x_1(t))^{\text{T}}, f_2(x_2(t))^{\text{T}}, \ldots, f_n(x_n(t))^{\text{T}} \right]^{\text{T}},$$

is a nonlinear function of $x(t)$.

III. CONTROL OPTIMIZATION OBJECTIVE

In this section, we formulate the control problem as an optimization problem.

A. Optimization Objectives

The goals of the proposed control strategy include reaching consensus, reducing battery degradation cost, and restricting BES SoC and BES output power within their respective boundaries. This can be achieved by minimizing the following objective function:

$$J = \epsilon_1 J_{\text{cons}} + \epsilon_2 J_{\text{dc}} + \epsilon_3 J_{\text{soc}} + \epsilon_4 J_{\text{bes}} \tag{18}$$

where ϵ_1, ϵ_2, ϵ_3, and ϵ_4 are significance factors, and J_{cons}, J_{dc}, J_{soc}, and J_{bes} are different performance indicators corresponding to the goals mentioned above. Note that the influence of a performance indicator (or optimization goal) can be adjusted by changing the corresponding significance factor according to the demand.

We now explain each of the performance indicators in (18) one by one.

1) Consensus: The first goal of our proposed control strategy is to reach consensus on r_i, the ratio of BES output power to BES SoC among microgrids. It is not hard to see that the consensus requirement (14) is satisfied if

$$\lim_{t \to \infty} \mathbb{E}[e_1(t)] = \lim_{t \to \infty} \mathbb{E}[e_2(t)] = \ldots = \lim_{t \to \infty} \mathbb{E}[e_n(t)] = 0.$$

In this case, reaching consensus is converted to minimizing the consensus errors of all microgrids. The corresponding performance indicator can be calculated by

$$J_{\text{cons}} = \mathbb{E}\left[\int_0^T \sum_{i=1}^n (e_i(t))^2 \, \mathrm{d}t\right], \tag{19}$$

where T is the termination time. We will provide more details about T in Section V.

2) Battery Degradation Cost: The frequent charging and discharging of the BES could cause battery degradation. In order to prevent overcontol and prolong the lifetime of the BES, battery degradation cost is an important indicator to be minimized. Based on the works of [36, 37], battery degradation cost can be simply calculated as follows:

$$J_{\text{dc}} = \mathbb{E}\left[\int_0^T \sum_{i=1}^n \left(\eta_i P_i^{\text{BES}}(t)\right)^2 \, \mathrm{d}t\right]. \tag{20}$$

3) Overflow Penalty: The control may cause damage to BES devices if it results in an SoC out of the recommended range defined by (6). This overflow can be evaluated by the following penalty function [59]:

$$\begin{aligned}\phi_i^{\text{soc}}(t) \quad &= \mathbb{I}(SoC_i(t) \leq SoC_i^{\text{min}}) \\ &+ \mathbb{I}(SoC_i(t) \geq SoC_i^{\text{max}}),\end{aligned} \tag{21}$$

where $\mathbb{I}(\cdot)$ is the characteristic function defined by:

$$\mathbb{I}(X) = \begin{cases} 1 & \text{if event } X \text{ is true,} \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding performance indicator for SoC overflow penalty would be:

$$J_{\text{soc}} = \mathbb{E}\left[\int_0^T \sum_{i=1}^n \phi_i^{\text{soc}}(t) \mathrm{d}t\right]. \tag{22}$$

The BES can also be damaged if the BES output power $P_i^{\text{BES}}(t)$ exceeds the boundary in (4). A similar penalty function for BES power overflow can be defined as [59]:

$$\phi_i^{\text{bes}}(t) = \mathbb{I}(|P_i^{\text{BES}}(t)| \geq P_i^{\text{max}}). \tag{23}$$

The corresponding performance indicator for BES power overflow is calculated by:

$$J_{\text{bes}} = \mathbb{E}\left[\int_0^T \sum_{i=1}^n \phi_i^{\text{bes}}(t) \mathrm{d}t\right]. \tag{24}$$

B. Problem Formulation

After describing the system dynamics and defining the objective function, our consensus-based control problem can be formulated as the following optimization problem:

$$\begin{aligned} \min_u \quad & J \\ \text{Subject to:} \quad & (1), (2), (3), (10)(7), \\ & (8), (11) \\ & 0 < SoC_i(t) < 1, \\ & 0 \leq |P_i^{\text{BES}}(t)| \leq P_i^{\text{max}}. \end{aligned} \tag{25}$$

The above problem is solved when a $u(t) = u^*(t)$ that minimizes J is discovered. As we have stated in Section II-B, the update of $r_i(t)$ described by (11) is achieved by the change in the BES power $P_i^{\text{BES}}(t)$, which can be indirectly controlled by the control signal $u_i^{\text{MT}}(t)$ to the MT. That is to say, the control problem of (25) is solved a consensus coefficient matrix $K = K^*$ that minimizes J is acquired and then $u_i^{\text{MT}}(t)$ is accordingly decided.

Note that the consensus variables in many existing works are linear to system state $x_i(t)$. In our case, however, the consensus variable $r_i(t)$ is not linear to $x_i(t)$ since it has $SoC_i(t)$ on its denominator (see (9)). Thus, existing solutions like local adjacent [10] may not be suitable for our problem. Moreover, the nonlinearity of system (17) increases the difficulty of not only adopting existing solutions but also solving the control problem with some traditional methods.

IV. SOLVING VIA COMPRESSIVE SENSING

In this section, we design an algorithm to find a $u(t) = u^*(t)$ that optimizes the nonlinear stochastic control problem of (25) by combining compressive sensing with an iterative method. Note that the system described by (17) is stochastic due to the Wiener process term $W(t)$. According to [65], the Hermite polynomial expansion coefficients of random variables are usually sparse, which satisfies the premise of solving by compressed sensing. It has been shown by [66, 67] that the compressive sensing method has a significant advantage in improving the efficiency of tackling high dimensional stochastic systems against traditional methods, e.g, the MC method [47].

A. State Variable Recovery via Compressive Sensing

First of all, we describe the process of solving the state equation (17) for $x(t)$ using the compressive sensing method in Algorithm 1, which is the most important step in our proposed

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Algorithm 1 Compressive Sensing for State Equation (17)

a. Rewrite the Wiener process $W(t)$:

$$W(t) = \sum_{j=1}^{\infty} \xi_j \int_0^t \varphi_j(\tau)\mathrm{d}\tau,$$

where $\{\xi_j\}_{j=1}^{\infty}$ are independent identically distributed random variables selected from the standard normal distribution, and $\{\varphi_j\}_{j=1}^{\infty}$ is a set of orthonormal bases.

b. Perform polynomial chaos expansion for $x(t)$:

$$x(t, \boldsymbol{\xi}) = \sum_{j=1}^{\infty} c_j(t)\psi_j(\boldsymbol{\xi})$$

where $\{\psi_j(\boldsymbol{\xi})\}_{j=1}^{\infty}$ are Hermite polynomials, $\boldsymbol{\xi} = [\xi_1, \xi_2, \ldots, \xi_j, \ldots]^{\mathrm{T}}$, and $\{c_j(t)\}_{j=1}^{\infty}$ are the sparse coefficients to be determined.

c. Find sparse coefficients $\{c_j\}_{j=1}^{\infty}$ by solving the following problem:

$$\hat{\boldsymbol{c}} = \arg\min \|\boldsymbol{c}\|_1, \quad \text{s.t.} \quad \|X - \Psi\boldsymbol{c}\| \leq \epsilon, \qquad (26)$$

where X is the sample simulation results of $x(t)$ at each point of time t, Ψ is an information matrix formed by inserting the stochastic sample points of X into Hermite polynomials, and $\boldsymbol{c} = [c_1, c_2, \ldots, c_j, \ldots]^{\mathrm{T}}$ is the coefficient vector to be determined.

Algorithm 2 Compressive Sensing-Based Gradient Descent (CSGD) for Problem (25)

a. Initialize parameters for optimization:
   including the number of Hermite polynomial basis to truncate the expansion in Algorithm 1, step size $z > 0$, tolerance parameter γ, time step Δt, initial consensus coefficient matrix K^0.

b. Deduce the Hamiltonian system for problem (25):

$$\mathrm{d}x = \frac{\partial H}{\partial \lambda}, \mathrm{d}\lambda = -\frac{\partial H}{\partial x}, 0 = \frac{\partial H}{\partial K}. \qquad (27)$$

   where H is the following Hamiltonian function [72]:

$$H = J + \dot{\lambda}(t)\{(Ax(t) + f(x(t)) + Bu(t))\mathrm{d}t + S\mathrm{d}W(t)\},$$

   and $\lambda(t)$ is the conjugate state corresponding to $x(t)$.

c. Use Algorithm 1 to derive the state solution $x^0(t)$ for system (17).

d. Loop for $\ell = 1, 2, \ldots$:
   1) From $x^{\ell-1}(t)$ and $K^{\ell-1}$, use Algorithm 1 to determine the solution $\lambda^{\ell-1}$ of the co-state equation (27);
   2) From $\lambda^{\ell-1}$, $K^{\ell-1}$ and $x^{\ell-1}$, determine the set of steps $\frac{\partial H}{\partial K^{\ell-1}}$ from $\frac{\partial H}{\partial K}$;
   3) From $K^{\ell-1}$ and $\frac{\partial H}{\partial K^{\ell-1}}$, determine the new value K^ℓ

$$K^\ell = K^{\ell-1} - z^{\ell-1}\frac{\partial H}{\partial K^{\ell-1}};$$

   4) From K^ℓ, use Algorithm 1 to calculate $x^\ell(t)$;
   5) From K^ℓ and $x^\ell(t)$, determine the value of the objective function J^ℓ. If the relative error reach the requirement, stop and output $K^* = K^\ell$. Otherwise, continue with the next loop.

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method. Since the uncertainty and randomness in the power of load and PV are captured by Wiener process $W(t)$, we can use Hermite polynomials to expand $W(t)$ [68]. For the feasibility of the numerical calculation of Step a and b in Algorithm 1, the expansion expressions are usually truncated with limited number of terms. In Step c, the expansion coefficient $\boldsymbol{c}$ of state variable $x(t)$ is recovered by solving the ℓ_1-minimization problem [66] described by (26).

Note that the most significant advantage of compressive sensing is that variables can be accurately recovered using only a small group of sample points, thus reducing the calculation cost to a great extent. Suppose d is the number of stochastic polynomial bases used for the (truncated) expansion in Step a and b. Then the total number of sample points required by the accurate recovery via compressive sensing, denoted by N, is approximately $c\log^4 d$, where c is a constant (please refer to [69, 70] for a more detailed proof). We can see that N is asymptotically smaller than d. Once the number of samples grows beyond N, adding more samples will no longer improve the accuracy, and the compressive sensing algorithm is said to converge. In contrast, the traditional MC method is semi-order convergence, which is much slower.

The error of the variable recovery by Algorithm 1 mainly comes from two parts: the truncation error caused by the expansion of stochastic terms (Step a) and solving the ℓ_1-minimization problem (26) (Step c). According to the analysis of [69], the truncation error is fixed based on the choice of the polynomial bases, while the error from solving ℓ_1-minimization can be reduced when the coefficient vector $\boldsymbol{c}$ is sparser. Therefore, the sparser the stochastic terms are, the

more accurate the compressive sensing method is. Readers can refer to [67, 71] for more details of the convergence analysis of Algorithm 1.

B. Solving Stochastic Optimal Control via Compressive Sensing-Based Gradient Descent

After $x(t)$ is acquired, we then use CSGD to solve the stochastic optimal control problem (25), i.e., find a consensus coefficient matrix $K = K^*$ that minimizes the optimization objective function J. Details of the algorithm are given in Algorithm 2. Note that CSGD is an iterative algorithm (e.g., gradient descent) combined with Algorithm 1

In Algorithm 2, the Hamiltonian system (27) in Step b is the equivalent system of problem (25). We can derive the solution of this optimal control problem by finding the solution of its corresponding Hamiltonian equation. The compressive sensing method in Algorithm 1 is used to calculate the system state $x(t)$ and the corresponding conjugate state $\lambda(t)$ in Step c and d. This dramatically reduces the computational cost of solving high-dimensional problems compared with traditional methods. Other steps in Algorithm 2 are typical steps of gradient descent method.

As we mentioned in Section III-B, we can derive the optimal control signal $u(t) = u^*(t)$ once K^* is determined by combining (3), (11), and (8).

V. SIMULATION RESULTS

In this section, we provide a detailed analysis on the simulation results of the numerical solution to (25) acquired by the algorithms in the previous section. All the results are obtained in MATLAB R2020b run on a computer with Intel Core i7-7700 CPU and a single GPU card with 2 GB of graphic memory. In particular, the recovery process in Step c of Algorithm 1 is accomplished by MATLAB toolbox SPGL1.

Fig. 2. Topology of the microgrid network for simulation.

Our simulation considers the network of $n = 5$ microgrids connected as Fig. 2. The corresponding adjacent matrix $\mathcal{A}$ is

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

Although there is no substantial difficulty in extending our results to large-scale systems, we simply choose $n = 5$ for illustrative purpose. Note that the topology in Fig. 2 is representative as the number of connections of a microgrid ranges from 1 to $n - 1$. Table II shows the selection of constants and parameters for the simulation. The selection is based on [53, 73, 74]. Moreover, we choose $\epsilon_1 = \epsilon_2 = 2$, $\epsilon_3 = \epsilon_4 = 1$.

TABLE II
SELECTION OF CONSTANTS AND PARAMETERS

i	1	2	3	4	5
ρ_i^{Load}	28.4	27.1	26.5	28.8	27
ρ_i^{PV}	16.9	17.1	18.7	17	16.2
σ_i^{Load}	0.4	0.26	0.35	0.28	0.3
σ_i^{PV}	0.29	0.31	0.27	0.3	0.25
η_i^{in}	0.95	0.98	0.97	0.92	0.95
η_i^{out}	0.96	0.90	0.92	0.90	0.97
Q_i (kW·h)	110	140	95	160	72
SoC_i^{min}	0.17	0.25	0.2	0.15	0.3
SoC_i^{max}	0.72	0.86	0.75	0.85	0.9
P_i^{max} (kW)	175	210	200	273	300
ρ_i^{TM} (s)	15	18	12.7	18.3	21
k_i^{TM}	7.9	9.2	10	8.5	8.6

Let the termination time $T = 15$ s. Note that the observation time of practical projects often lasts for hours or days. In contrast, the modeling of this paper is based on differential equations that capture system dynamics in a relatively shorter time frame. Although there should be no essential difficulty to extend our work to a longer period by continuously repeating the simulation in a row, we choose not to do so. The main reason is that this repetition could cause the parameters and constants in Table II to vary, and they need to be remeasured accordingly. This is the work of researchers specialized in parameter measurement, which is beyond the scope of this paper.

Fig. 3 shows an example of the power change simulation in the load and the PV of MG_3 obtained by (1) and (2) with the parameters in Table II. It can be seen that both curves present significant zigzags. This is caused by diffusion terms $W_i^{\text{Load}}(t)$ and $W_i^{\text{PV}}(t)$, which can accurately describe the instantaneous drastic changes caused by random disturbances or unexpected operation.
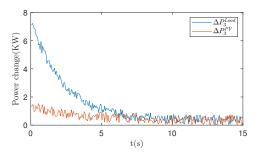
Fig. 3. Load and PV power changes in MG_3

A. Convergence and Consensus

The convergence of the algorithm can be seen from Fig. 4 that shows the decrease of the objective function J with the number of iterations of Algorithm 2. It can be seen that J falls dramatically in the first 10 iterations and then asymptotically goes to 0. Under the given error accuracy $\gamma = 0.00001$, the algorithm converges after about 35 iterations.
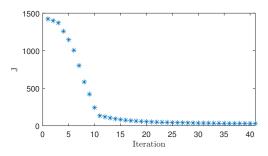
Fig. 4. Convergence of the objective function

Fig. 5 shows the convergence of the consensus variable r_i among different microgrids. It indicates that consensus is reached very fast (within 8 seconds). Note that the strict consensus in (12) cannot be achieved because of diffusion terms $W_i^{\mathrm{Load}}(t)$ and $W_i^{\mathrm{PV}}(t)$. As a result, the curves fluctuate slightly around the consensus value ($r^* \approx 0.02$).
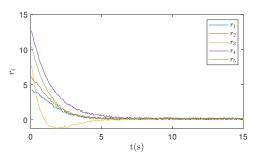

Fig. 5. Consensus on r_i among microgrids.

The data in Table III shows an 89.12% reduction in the computation time cost of our compressive sensing-based method provided in Section IV compared with that of the tradition MC method [47].

TABLE III
COMPARISON OF COMPUTATION TIME COST BETWEEN CSGD AND THE
MC METHOD

Method	CSGD	MC
Time Cost (s)	312.87	2892.16

In addition, we also compare our CSGD method to the traditional MC method for the analysis of stability. When the scale of sample points reaches a certain level, our CSGD method and MC method yield the same results. The data in Table IV show that the error of CSGD gradually converges as the number of sample points increases. It can be seen that our CSGD algorithm with 100 samples can achieve the same accuracy as the traditional MC method with 2000 samples, which is also the reason we choose compressive sensing against MC.
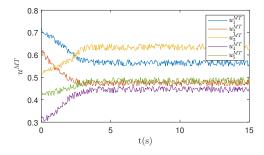
TABLE IV
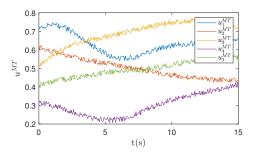COMPARISON OF THE ACCURACY BETWEEN CSGD AND THE MC
METHOD WITH DIFFERENT NUMBERS OF SAMPLE POINTS

Sample Number	Error of MC ($\times 10^{-2}$)	Error of CSGD ($\times 10^{-2}$)
20	3.43	8.91
40	2.95	1.23
60	2.43	0.46
80	1.56	0.04
100	1.21	0.03
1000	0.32	0.03
2000	0.03	0.03

Note that the control of the BES power is indirectly achieved by applying control to MTs. Fig. 6 compares the control signals of MT with and without consensus. It can be seen

that the control signals with consensus are limited within a smaller range. This can mitigate the damage to MTs caused by overcontrol [54].

(a) Control Signal of MT with consensus

(b) Control Signal of MT without consensus

Fig. 6. Comparison of control signals (with consensus vs. without consensus).

B. Management of Battery Energy Storage Devices

The comparison of the output powers and SoCs of the BESs with and without control is demonstrated by Fig. 7. We can see from Fig. 7a that the output powers of all BESs are reduced to a much lower level when under control compared with Fig. 7b. This remarkable contrast results from $J_{\mathrm{degradation}}$, the performance indicator for battery degradation cost, in our control optimization objective. The data in Table V shows a significant 74.24% reduction on average in battery degradation cost in each microgrid when applying our control strategy compared to the case without controlling degradation cost.

TABLE V
COMPARISON OF BATTERY DEGRADATION COSTS J_{DC} (WITH
DEGRADATION COST CONTROL VS. WITHOUT DEGRADATION COST
CONTROL)

	MG_1	MG_2	MG_3	MG_4	MG_5
With degradation cost control	0.0729	0.2025	0.5658	5.2239	1.7842
No degradation cost control	1.0305	1.9256	3.8677	7.6654	6.2784
Reduction (%)	92.93	89.48	85.37	31.85	71.58

(a) Output power of BES with control

(b) Output power of BES without control

(c) SoC of BES with control
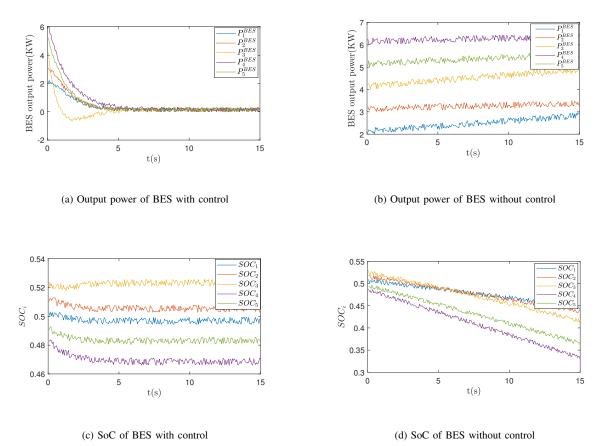

(d) SoC of BES without control

Fig. 7. Comparison of output power and SoC of BES (with control vs. without control).

Since the output powers of BESs are greatly reduced after control, the fluctuation of the SoCs in Fig. 7c is relatively gentle. In contrast, when there is no control, the SoCs are not stabilized and will continue to change after 15 seconds (as shown by Fig. 7d).

VI. CONCLUSION

Consensus-based control helps to achieve global optimization in a cooperative way and is widely used in the distributed control in energy systems. Our proposed optimal control strategy chooses to synchronize the ratio of BES output power to BES SoC among multiple microgrids. It also considers minimizing battery degradation cost, which is seldom addressed in existing consensus-based control methods. We use compressive sensing method to solve the nonlinear control problem, which is more efficient than the traditional MC method.

Although consensus-based control has the advantage of fast and strong convergence to the global optimality, it has a series of drawbacks caused by its state sharing through communication links. On the one hand, sharing too many state variables will lead to the surge of information transmission workload. This problem can be alleviated by reducing the number of shared variables. On the other hand, the real-time state detection on neighbor microgrids also brings a

high volume of communication. In practice, discrete state sampling [75] could be a potential solution to this problem and will be considered in our future work. Our future work will also consider the more complex system that combines renewable energy power generation devices with traditional power generation devices (e.g., microturbines or diesel engine generators).

As the concept of EI also emphasizes the complementation of multiple energy sources, our future work will try to extend the result of this paper to the dispatch, coordination, and regulation of multi-energy systems [76, 77]. Moreover, as the interaction with energy consumers becomes more and more pervasive in EI systems [78], there are many scenarios that need include economic concerns, e.g., economic dispatch, demand response, energy trading. In this case, the control objectives might need to consider the competition between energy supply and energy consumption [79, 80]. This is also a possible direction for our future work.

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