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Voltage Control for Uncertain Stochastic Nonlinear System with Application to Energy Internet: Non-fragile Robust H_∞ Approach

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Abstract

In this paper, a robust control method is used to intelligently regulate the voltage deviation for the DC microgrid (MG) in an off-grid mode. In order to cope with the uncertain and nonlinear nature of the power exchange in the scenario of energy internet (EI), a new mathematical model is proposed, where the state equation is expressed by a class of nonlinear stochastic differential equations (SDEs) with parameter uncertainties appearing in system coefficients and the controller. Specifically, the nonlinearities appear in both drift and diffusion parts of such SDEs. Our aim is to design a controller such that the effects of the modelling uncertainty and the external disturbance to the DC bus voltage of the off-grid MG are minimised. Mathematically, this can be regarded as the problems of solving the non-fragile robust stochastic stabilization and non-fragile robust H_∞ control, both of which can be solved via the linear matrix inequality (LMI) approach sufficiently. We emphasize the new control system proposed with its solution can also be applied into the fields of industry, economics, etc. Based on real world data, numerical results are illustrated, and simulations show that our target is achieved.

Keywords: Stochastic System; Robust Control; H_∞ Control; Energy Internet.

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1. Introduction

Nowadays human beings are facing global challenges such as environmental pollution, climate change, oil crisis, etc. Improvement in technologies of renewable energy and modern information and communication technology (ICT) are enlightened to deal with such issues. Rifkin proposed a new energy system where energy and information are integrated, and it was named as energy internet (EI) for the first time [1]. Afterwards, there have been a variety of research works about EI regarding its infrastructure, features, architecture, energy router (ER) and other related aspects, see, e.g., [2], [3], [4], [5]. In [2] it is pointed out that EI can be viewed as the version 2.0 of smart grids, and it is concluded that EI is an internet based wide area network that integrates information and energy bi-directly, whereas the main power grid is regarded as the backbone network and the microgrid (MG) or distributed energy resources (DERs) as local area network.

When we consider EI for a specific area, whether it is large or small, MG always exists in it as a core element. There is one special operation mode of MG especially for the areas far away from the urban, namely, islanded (or off-grid) MG. One of the main reasons for utilizing such mode is that power delivery may be costly. For real installed islanded MG, reader can refer to e.g., [7], [8], [9]. In this paper, we consider a scenario in which EI is operated in an islanded mode, i.e, EI of such type is remote as well as off-grid, and all the power consumption within that area can only be supplied by DERs. Moreover, we assume that fossil fuels are excluded in this particular scenario of EI, due to the delivery cost as well as environmental issues.

In EI a typical infrastructure of the DERs is composed of the following elements: wind turbine generator (WTG), photovoltaic panel (PV), fuel cell (FC), micro turbine (MT), battery energy storage (BES) and flywheel energy storage (FES) [5]. In a real world application of EI, it is the ER that links energy generators, energy storage devices and the loads, so that energy from different sources are fused and interacted [2]. In the same literature, it is emphasized that one of the key functions of ER is energy balancing, i.e., if exceeded amount of power is produced beyond the requirement of the local loads, the redundant power could be delivered by ER into the grid or other MGs, so that neighbour loads could consume power. Inversely, when local area is short of power production, ER can introduce power from the grid wherever there is a surplus.

On the other hand, as core components of EI, although DER units have a number of environmental and economic benefits [6], they have shortcomings such as low inertia, uncertainty, dynamic complexity, nonlinearity and intermittence [10], all of which bring challenges to EI. For example, wind and solar radiation normally cannot be predicted precisely and they actually vary stochastically [11], [12]. Thus WTG and PV could introduce uncertainty into the power system of EI. When we model the loads in electricity power systems, we take the sample from a large number of individual and independent customer devices. The usage of power system loads depends on various factors such as time, geography, etc., hence the loads vary stochastically as well [13]. In the off-grid EI, the total power generation of the distributed resources supporting the demand side includes the power output from WTG, PV, FC, MT, with power exchange from BES, FES and ER. Naturally, a mismatch between power supply and load will cause voltage deviation and even blackout of the whole EI. In this case, robust control schemes are expected to be considered for the power system in EI such that robust performance and robust stability is achieved.

When there exist exogenous disturbances in a system, we normally design a control law such that the effect of the disturbances is eliminated efficiently, and this is known as H_∞ control method. In classic H_∞ theory, the deterministic H_∞ problems are considered in frequency domain, where H_∞ norm is defined by a norm of the rational transfer matrix. The shortcoming of the frequency domain approach is that H_∞ theory can only be applied to linear deterministic systems. Such restriction is released by considering the time domain approach (also known as state-space formulation), which allows H_∞ theory to be applied to either nonlinear or stochastic systems [14], [15].

For power systems, robust control method has been widely used to reduce the influence on a variety of perturbations, see, e.g., [10], [16], [17], [18], [19], [20] , and the references therein, among which robust H_∞ approach is used in [10], [18], and [20]. It is notable that most literatures regarding the applications of H_∞ control to power systems adopt the frequency domain approach, where the power systems are described by linear ordinary differential equations (ODEs) in the deterministic case. Regarding most of the nonlinear power systems in reality, researchers normally approximate them into linear cases. For example, in the field of MG, authors of [21] transform the nonlinear heterogeneous dynamics of distributed generators into linear ones via input-output feedback linearisation. In addition, some stochastic models are approximated into deterministic systems, for the sake of computational simplicity in either simulations or experiments.

In the scenario of the EI we require power quality to be guaranteed. Nonetheless, in reality it is very possible that a tiny bit of perturbation on the controller parameter may lead to disability of the whole power system [5]. Hence, we are required to design a controller that is robust with respect to (w.r.t.) variations of its parameters. This is referred to as the non-fragile robust control. Recently, non-fragile robust H_∞ control problems are very popular. For example, [22] deals with neutral system with time-varying delays. Delay-dependent non-fragile robust H_∞ filtering of T-S fuzzy time-delay systems was investigated by [23].

When dealing with robust control problems for power systems, most of the existing works mentioned above transform the physical models into some mathematical systems of linear ODEs, then classic robust control algorithms and techniques are applied directly to obtain the desired results. Afterwards, numerical results or simulations are worked out by software, e.g., Matlab, power system computer aided design (PSCAD). Here we emphasise that the above research approach has several disadvantages. Most of the works do not take modelling errors of the power system into consideration. Meanwhile, excessive approximation is unavoidable to obtain the ideal mathematical model. Especially for systems studied in frequency domain, the existing stochasticities and nonlinearities cannot be considered, which is conservative and restrictive. In addition, from the best of authors' knowledge there is few research paper in the field of power systems that considers the non-fragile robust H_∞ controller.

In this paper, we investigate the system of an off-grid DC MG under the scenario of EI. Particularly, we assume that within a certain area, there are only two DC MGs interconnected with each other via their ERs. For any of these DC MGs, our goal is to find a non-fragile robust H_∞ controller such that the DC bus voltage is robustly stochastically stabilizable with a prescribed disturbance attenuation level. It is noted that such typical model can be extended into a more general version, where more than two DC MGs are allowed to be mutually interconnected. We highlight the importance of our work by revivifying the characteristics of the original physical EI system via the following approaches. When formulating the mathematical problem we adopt time domain approach, where the state equation is described by a class of nonlinear SDEs. We emphasize that nonlinearities appear in both drift and diffusion parts, and we assume they follow a class of norm bounded conditions. Besides, we allow parameter uncertainties in the system coefficients as well as the controller, due to some uncertain factors. It is notable that the non-fragile robust H_∞ control problem for such particular stochastic nonlinear system has not been considered before, due to its complexity. Then we solve our problem via LMI approach, with two theorems obtained in the main result. For this particular stochastic nonlinear system, the desired controller is linear with state, which is a rare case when the system itself is nonlinear. Different from most literatures in the field of power system that addressing computational algorithms, this paper places emphasis on providing analytical results. It is worth mentioning that our new control system proposed can be applied not only in the system of EI, but also in a variety of other industrial and economic scenarios, such as flight control, bond pricing, etc.

The rest of the paper is organised as follows. In Section 2, the dynamic model of the off-grid DC MG is presented, and in Section 3, we formulate our robust control problem mathematically and introduce some preliminaries of robust H_∞ control problem. In Section 4, we derive sufficient conditions such that the system is robustly stochastically stabilizable. In Section 5, we solve our non-fragile robust H_∞ control problem. In Section 6, we provide some numerical simulations. In Section 7, we conclude our work.

Notation:
Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the n-dimensional Euclidean space and the set of all $m \times n$ real matrices, respectively. The superscript $'$ represents the transpose. The asterisk $*$ in a matrix is used to represent the term which is induced by symmetry. For symmetric matrices X and Y, the notation $X \geq Y$ ($X > Y$) means that the matrix $X - Y$ is positive semi-definite (positive definite). $\mathbf{I}_m$ is the m-dimensional identity matrix. $\mathbb{E}$ represents the mathematical expectation operator w.r.t. some probability measure $\mathbb{P}$. $L_2[0,\infty)$ is the space of square-integrable vector function over $[0,\infty)$. $|\cdot|$ denotes the Euclidean vector norm. $||\cdot||$ refers the usual $L_2[0,\infty)$ norm. $\{\Omega, \mathcal{F}, \mathbb{P}\}$ stands for a probability space in which Ω is the sample space and $\mathcal{F}$ is the σ-algebra of subsets of sample space. Matrices are assumed to be compatible for algebraic operations, if not explicitly stated.

2. Energy Internet Modelling

In this paper we consider the scenario of EI where the MGs are in an off-grid mode, i.e., the main power grid is not considered. Particularly, we only consider two interconnected DC MGs within a certain area. We assume that there exists an ER in each DC MG. The ERs can be regarded as intermediaries that are used to connect both DC MGs. We adopt the following the energy routing strategy, see, e.g., [24] and the references therein. When the local DC MG is short of energy (especially when the local energy storage is almost empty), its ER can be used to deliver/transfer energy from the other DC MG to the local one. When the local DC MG has energy surplus (especially when the local energy storage is almost full), the abundant energy can be transferred to the other DC MG via

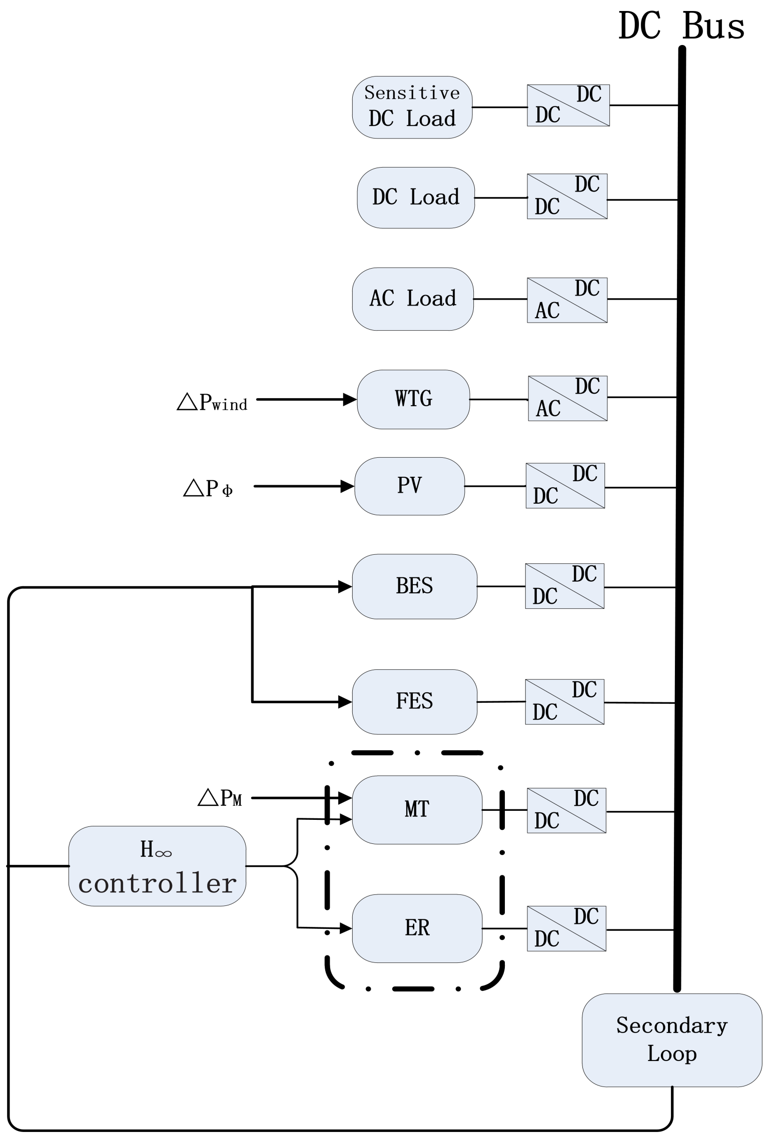

Figure 1: DC MG Voltage Control Model

the ER.

Our attention is paid on one single DC MG, in which stability and the H_∞ performance w.r.t. the DC bus voltage deviation are desired to be achieved. A typical DC MG is analysed in Fig. 1, with the H_∞ controller demonstrated.

As shown in Fig. 1, WTG, PV, BES, FES, MT, ER and loads are connected to the DC bus. Since the power generation by WTG and PV mainly depends on the weather, normally they are not used for voltage regulation. The H_∞ controller is set in the MT and the ER. We denote ΔP_{Wind} and ΔP_φ as wind power change and solar radiation power change, respectively. When energy is transferred via ERs, the power output change of ER in one DC MG is unavoidably affected by the power output from the other DC MG. The power output transferred from the other DC MG might be unstable as well as time varying, and it can be regarded as disturbance w.r.t. the power output change of the ER in the local DC MG. We denote such power disturbance as ΔP_M. The power balance in Fig. 1 is reached if the following formula is held:

$$\Delta P_{WTG} + \Delta P_{PV} + \Delta P_{MT} + \Delta P_{BES} + \Delta P_{FES} + \Delta P_{ER} + \Delta P_{Load} = 0, \quad (1)$$

where ΔP_{WTG}, ΔP_{PV}, ΔP_{MT}, ΔP_{BES}, ΔP_{FES}, ΔP_{ER} and ΔP_{Load} stand for the power output change of WTG, PV, MT, BES, FES, ER and load. The modelling of MG has been proposed in a variety of works, see, e.g., [10], [16], [17], [21], [25], [26], [27], [28], etc., where the systems are described by linear ODEs. In the next section, considering the stochastic property of wind, solar radiation and loads, we propose the mathematical model of such DC MG, formed by a group of ODEs and SDEs.

3. Mathematical Problem Formulation

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a given filtered complete probability space, where there exist two independent scalar Brownian motions $W_{Wind}(t)$ and $W_\varphi(t)$ representing for stochasticity caused by wind and solar radiation, respectively. First, if we exclude the parameter uncertainty and system nonlinearity, then the system of

DC MG can be roughly formulated as follows,

$$\begin{cases} d\Delta P_{WTG} = \big(-\frac{1}{T_{WTG}}\Delta P_{WTG} + \frac{1}{T_{WTG}}\Delta P_{Wind}\big)dt \\ \qquad\qquad + \big(-\frac{1}{S_{WTG}}\Delta P_{WTG} + \frac{1}{S_{WTG}}\Delta P_{Wind}\big) \times dW_{Wind}(t), \\ d\Delta P_{PV} = \big(-\frac{1}{T_{PV}}\Delta P_{PV} + \frac{1}{T_{PV}}\Delta P_{\varphi}\big)dt \\ \qquad\qquad + \big(-\frac{1}{S_{PV}}\Delta P_{PV} + \frac{1}{S_{PV}}\Delta P_{\varphi}\big)dW_{\varphi}(t), \\ d\Delta P_{ER} = \big(-\frac{1}{T_{ER}}\Delta P_{ER} + \frac{1}{T_{ER}}u + \frac{1}{T_{ER}}\Delta P_{M}\big)dt, \\ d\Delta P_{MT} = \big(-\frac{1}{T_{MT}}\Delta P_{MT} + \frac{1}{T_{MT}}u\big)dt, \\ d\Delta P_{BES} = \big(-\frac{1}{T_{BES}}\Delta P_{BES} + \frac{1}{T_{BES}}\Delta V\big)dt, \\ d\Delta P_{FES} = \big(-\frac{1}{T_{FES}}\Delta P_{FES} + \frac{1}{T_{FES}}\Delta V\big)dt, \\ d\Delta V = \big(-\frac{1}{\alpha}\Delta V + \frac{1}{\beta}\Delta P_{Load}\big)dt, \end{cases} \tag{2}$$

where T_{WTG}, T_{PV}, T_{ER}, T_{MT}, T_{BES} and T_{FES} are the time constants of WTG, PV, ER, MT, BES and FES, respectively. In (2), S_{WTG}, S_{PV}, α and β are system parameters, which can be measured in engineering practice.

Since our purpose is to find a controller such that the voltage regulation problem is solvable, power balance described in (1) is expected to be achieved. Then ΔP_{Load} in (2) can be substituted by $-(\Delta P_{WTG} + \Delta P_{PV} + \Delta P_{MT} + \Delta P_{BES} + \Delta P_{FES} + \Delta P_{ER})$ according to (1). We define the DC bus voltage deviation ΔV as the controlled output of system (2). Then (2) can be rewritten as a linear stochastic state-space control system,

$$\begin{aligned} dx(t) &= [Ax(t) + Bu(t) + Cv(t)]dt + [D_1x(t) + E_1v(t)]dW_{Wind}(t) \\ &\quad + [D_2x(t) + E_2v(t)]dW_{\varphi}(t) \end{aligned} \tag{3}$$

$$z(t) = Fx(t), \tag{4}$$

where $x(t) = \begin{bmatrix} \Delta P_{WTG} \\ \Delta P_{PV} \\ \Delta P_{FC} \\ \Delta P_{MT} \\ \Delta P_{ES} \\ \Delta P_{ER} \\ \Delta V \end{bmatrix}$ is the system state, $u(t)$ is the control input, $v(t) = \begin{bmatrix} \Delta P_{Wind} \\ \Delta P_{\varphi} \\ \Delta P_{M} \end{bmatrix}$ is the disturbance input, and $z(t) = \Delta V$ is the controlled output.

Since our purpose of modelling the DC MG is to obtain a mathematical control system, where H_∞ control method can be applied to solve the robust control problem directly, then from the perspective of solving a mathematical problem we prefer to assuming that $W_{Wind}(t) = W_{\varphi}(t) \triangleq W_1(t)$ for simplicity, but without loss of generality. Also, we define that $D \triangleq D_1 + D_2$, $E \triangleq E_1 + E_2$. Then (3) can be rewritten as

$$dx(t) = [Ax(t) + Bu(t) + Cv(t)]dt + [Dx(t) + Ev(t)]dW_1(t), \tag{5}$$

where system coefficients are as follows,

$$A = \begin{bmatrix} -1/T_{WTG} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1/T_{PV} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1/T_{ER} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1/T_{MT} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1/T_{BES} & 0 & 1/T_{BES} \\ 0 & 0 & 0 & 0 & 0 & -1/T_{FES} & 1/T_{FES} \\ -1/\beta & -1/\beta & -1/\beta & -1/\beta & -1/\beta & -1/\beta & -1/\alpha \end{bmatrix},$$

$$
B = \begin{bmatrix} 0 \\ 0 \\ {}^{1}/T_{ER} \\ {}^{1}/T_{MT} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} {}^{1}/T_{WTG} & 0 & 0 \\ 0 & {}^{1}/T_{PV} & 0 \\ 0 & 0 & {}^{1}/T_{ER} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}',
$$

$$
D = \begin{bmatrix} -{}^{1}/S_{WTG} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -{}^{1}/S_{PV} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad E = \begin{bmatrix} {}^{1}/S_{WTG} & 0 & 0 \\ 0 & {}^{1}/S_{PV} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.
$$

We claim that if we could solve the H_∞ control problem in forms of (5), then the same technique could be applied to solve the same problem represented in forms of (3) without essential difficulty.

The stochastic power system dynamics have been investigated since the 1980s, see, e.g., [29], [30], [31], [32]. When numerous uncertainties are involved into the system, it has attracted researchers' attention in recent years, see, e.g., [33]. Due to the modelling errors and various uncertainty factors in our real power system of EI, some parameters of system (5) cannot be measured accurately. Besides, DC MG itself has nonlinear dynamic complexity. By taking into account the parameter uncertainty and the unavoidable nonlinearities, we suggest the following model that would be more suitable to describe the real physical mechanism of DC MG. Let $W_2(t)$ be a one-dimensional standard Brownian motion independent of $W_1(t)$, both of which be defined in the filtered complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$. We consider a class of uncertain stochastic nonlinear system as follows:

$$\begin{aligned} dx(t) &= [(A+\Delta A(t))x(t) + Bu(t) + Cv(t) + f(x,t)]dt \\ &\quad +[(D+\Delta D(t))x(t) + Ev(t)]dW_1(t) + g(x,t)dW_2(t), \end{aligned} \tag{6}$$

$$z(t) = Fx(t). \tag{7}$$

In system (6)-(7), A, B, C, D, E and F are known real constant matrices, whereas unknown matrices $\Delta A(t)$ and $\Delta D(t)$ are time-varying parameter uncertainties, which are assumed to be of the following form

$$\begin{bmatrix} \Delta A(t) & \Delta D(t) \end{bmatrix} = M_1 U_1(t) \begin{bmatrix} N_a & N_d \end{bmatrix}, \tag{8}$$

where M_1, N_a and N_d are known real matrices and $U_1(\cdot)$ is an unknown time-varying matrix function that satisfies

$$U_1(t)' U_1(t) \leq I, \tag{9}$$

for all t. When both (8) and (9) hold, $\Delta A(t)$ and $\Delta D(t)$ are said to be admissible. Such uncertainty structure has been used by many authors, see, e.g. [34], [35], [36], etc. The term $f(x(t))$ is denoted as nonlinear uncertainty satisfying the following gain bounded condition:

$$f(x,t) = T\delta(x,t), \tag{10}$$

where

$$||\delta(x,t)|| \leq ||Lx(t)||. \tag{11}$$

The term $g(x(t))$ is an unknown nonlinear function describing the stochastic nonlinearity of the following form

$$g(x,t)' g(x,t) \leq x(t)' G' G x(t). \tag{12}$$

Note that the above nonlinearity of $g(x(t))$ has been used in e.g. [37]. Here T, L and G are known real constant matrices with appropriate dimensions. We emphasize that both $f(x(t))$ and $g(x(t))$ are assumed to be locally bounded and locally Lipschitz continuous with $f(x(0)) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}'$ and $g(x(0)) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}'$. In this sense, the local existence and uniqueness of strong solutions to the SDE (6) are assured [38], [39], [40].

Due to the existence of perturbations on the control gain, the non-fragile controller has to be considered, which is of the following form,

$$u(t) = (K + \Delta K(t))x(t), \tag{13}$$

where K is the nominal control gain matrix, whereas $\Delta K(t)$ denotes the control gain variation, whose form is similar to (8), i.e.,

$$\Delta K(t) = M_2 U_2(t) N_k, \tag{14}$$

where M_2 and N_k are known real matrices and $U_2(\cdot)$ is an unknown time-varying matrix function that satisfies

$$U_2(t)' U_2(t) \leq I, \tag{15}$$

for all t.

Next, we recall some definitions of robust stability, stabilization and H_∞ control.

Definition 1. *(See, e.g., [34].) The system in (6) and (7) with $u(t) = 0$ and $v(t) = 0$ is said to be mean-square asymptotically stable if*

$$\lim_{t \to \infty} \mathbb{E}|x(t)|^2 = 0$$

for any initial conditions.

Definition 2. *(See, e.g., [41].) The uncertain stochastic system in (6) and (7) is said to be robustly stochastically stable if the system associated to (6) and (7) with $u(t) = 0$ and $v(t) = 0$ is mean-square asymptotically stable for all admissible uncertainties $\Delta A(t)$ and $\Delta D(t)$.*

Definition 3. *(See, e.g., [42].) Given a scalar $\gamma > 0$, the stochastic system from (6) to (7) with $u(t) = 0$ is said to be robustly stochastically stable with* *disturbance attenuation γ if it is robustly stochastically stable and under zero initial conditions, $\|z(t)\|_2 < \gamma\|v(t)\|_2$ for all non-zero $v(t)$ and all admissible uncertainties $\Delta A(t)$ and $\Delta D(t)$, where*

$$\|z(t)\|_2 = \left(\mathbb{E}\left\{\int_0^\infty |z(t)|^2 dt\right\}\right)^{\frac{1}{2}}.$$

Lemma 1. *(See, e.g., [43].) Let matrices $\mathsf{M} = \mathsf{M}'$, N, and $\mathsf{R} = \mathsf{R}'$ be given with appropriate dimensions. Then the following conditions are equivalent:*

(i) $\mathsf{M} - \mathsf{N}\mathsf{R}^{-1}\mathsf{N}' \geq 0$ *and* $\mathsf{N}(I - \mathsf{R}\mathsf{R}^{-1}) = 0$, $\mathsf{R} \geq 0$,

(ii) $\begin{bmatrix} \mathsf{M} & \mathsf{N} \\ \mathsf{N}' & \mathsf{R} \end{bmatrix} \geq 0$,

(iii) $\begin{bmatrix} \mathsf{R} & \mathsf{N}' \\ \mathsf{N} & \mathsf{M} \end{bmatrix} \geq 0$.

Lemma 2. *(See, e.g., [44].) Let $\mathcal{A}, \mathcal{D}, \mathcal{S}, \mathcal{W}$ and F be real matrices of appropriate dimensions such that $\mathcal{W} > 0$ and $F'F \leq I$. Then we have the following.*

For scalar $\epsilon > 0$ and vectors $x, y \in \mathbb{R}^n$

$$2x'\mathcal{D}F\mathcal{S}y \leq \epsilon^{-1}x'\mathcal{D}\mathcal{D}'x + \epsilon y'\mathcal{S}'\mathcal{S}y.$$

For any scalar $\epsilon > 0$ such that $\mathcal{W} - \epsilon\mathcal{D}\mathcal{D}' > 0$

$$(\mathcal{A} + \mathcal{D}F\mathcal{S})'\mathcal{W}^{-1}(\mathcal{A} + \mathcal{D}F\mathcal{S}) \leq \mathcal{A}'(\mathcal{W} - \epsilon\mathcal{D}\mathcal{D}')^{-1}\mathcal{A} + \epsilon^{-1}\mathcal{S}'\mathcal{S}.$$

Our goal is to find the value of K in (13), such that our non-fragile robust stabilization problem and non-fragile robust H_∞ control problem are solved. In this sense, the effects of the modelling uncertainty and the external disturbance to the DC bus voltage of the DC MG are minimised.

4. Non-fragile Robust Stochastic Stabilization

In this section, we design a non-fragile state feedback controller for the uncertain stochastic nonlinear system (6), such that system (6) is robustly stochastically stabilizable. The problem is solved by the LMI approach.

Theorem 1. *Consider the uncertain nonlinear SDE (6) with $v(t) = 0$. Given a symmetric matrix $N > 0$, our system (6) with $K = YX^{-1}$ is robustly stochastically stabilizable if there exist scalars $\epsilon_1 > 0$, $\epsilon_2 > 0$, $\epsilon_3 > 0$, matrices Y, and symmetric positive definite matrix $X > 0$, such that the LMI (16) holds,*

$$\begin{bmatrix} \bar{\Gamma}_1 & M_1 & XN_a' & BM_2 & XN_k' & T & XL' & XG' & XN_d' & XD' \\ \star & -\epsilon_1^{-1}I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & -\epsilon_2^{-1}I & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & -\epsilon_2 I & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -N & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -N^{-1} & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & -X & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & -\epsilon_3 I & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & \bar{\Gamma}_2 \end{bmatrix} < 0, \tag{16}$$

where $\bar{\Gamma}_1 = AX + BY + XA' + Y'B'$, $\bar{\Gamma}_2 = \epsilon_3 M_1 M_1' - X$.

Proof. Substituting the non-fragile state feedback controller (13) into system (6) with $v(t) = 0$ and the uncertainty structure (8) with (14), we have the closed-loop system as

$$\begin{aligned} dx(t) &= [\bar{A}x(t) + M_1 U_1(t) N_a x(t) + BM_2 U_2(t) N_k x(t) + f(x(t))]dt \\ &\quad + [Dx(t) + M_1 U_1(t) N_d x(t)]dW_1(t) + g(x(t))dW_2(t), \end{aligned} \tag{17}$$

where $\bar{A} = A + BK$. Let $P = X^{-1}$, then from (16) it is easy to get

$$P^{-1} - \epsilon_2 MM' > 0. \tag{18}$$

We define the Lyapunov function as

$$V(x(t), t) = x(t)'Px(t). \tag{19}$$

According to Itô's formula, we have

$$\begin{aligned} dV(x(t), t) &= \mathcal{L}V(x(t), t)dt + 2x(t)'Pg(x(t))dW_2(t) \\ &\quad + 2x(t)'P[Dx(t) + M_1 U_1(t) N_d x(t)]dW_1(t), \end{aligned} \tag{20}$$

where the operator $\mathcal{L}V(x(t),t)$ is

$$\begin{aligned}\mathcal{L}V(x(t),t) &= 2x(t)'P[M_1U_1(t)N_ax(t)+BM_2U_2(t)N_kx(t)\\ &\quad +\bar{A}x(t)+f(x(t))]+g(x(t))'Pg(x(t))\\ &\quad +x(t)'[D+M_1U_1(t)N_d]'P[D+M_1U_1(t)N_d]x(t). \qquad (21)\end{aligned}$$

By Lemma 2 and (10) to (12), we have the following matrix inequalities.

$$2x(t)'PM_1U_1(t)N_ax(t)\le x(t)'(\epsilon_1PM_1M_1'P+\epsilon_1^{-1}N_a'N_a)x(t), \qquad (22)$$

$$2x(t)'PBM_2U_2(t)N_kx(t)\le x(t)'(\epsilon_2PBM_2M_2'B'P+\epsilon_2^{-1}N_k'N_k)x(t), \qquad (23)$$

$$2x(t)'Pf(x(t))\le x(t)'(PTN^{-1}T'P+L'NL)x(t), \qquad (24)$$

$$\begin{aligned}&[D+M_1U_1(t)N_d]'P[D+M_1U_1(t)N_d]\\ \le\ & D'(P^{-1}-\epsilon_3M_1M_1')^{-1}D+\epsilon_3^{-1}N_d'N_d, \qquad (25)\end{aligned}$$

$$g(x(t))'Pg(x(t))\le x(t)'G'PGx(t), \qquad (26)$$

where N is an invertible matrix with appropriate dimension. Then (21) can be rewritten as

$$\mathcal{L}V\le x(t)'\Theta x(t), \qquad (27)$$

where

$$\begin{aligned}\Theta &= P(A+BK)+(A+BK)'P+\epsilon_1PM_1M_1'P+\epsilon_1^{-1}N_a'N_a\\ &\quad +\epsilon_2PDM_2M_2'D'P+\epsilon_2^{-1}N_k'N_k+PTN^{-1}T'P+L'NL+G'PG\\ &\quad +\epsilon_3^{-1}N_d'N_d+D'(P^{-1}-\epsilon_3M_1M_1')^{-1}D. \qquad (28)\end{aligned}$$

Let us pre and post multiply LMI (16) by

$$diag(P,I,I,I,I,I,I,I,I,I),$$

then we have the following matrix inequality

$$
\begin{bmatrix}
\Gamma_1 & PM_1 & N_a' & PBM_2 & N_k' & PT & L' & G' & N_d' & D' \\
\star & -\epsilon_1^{-1}I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\star & \star & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\star & \star & \star & -\epsilon_2^{-1}I & 0 & 0 & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & -\epsilon_2 I & 0 & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & -N & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & -N^{-1} & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & -P^{-1} & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & -\epsilon_3 I & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \Gamma_2
\end{bmatrix}
< 0, \quad (29)
$$

where $\Gamma_1 = P(A + BK) + (A + BK)'P$, $\Gamma_2 = \epsilon_3 M_1 M_1' - P^{-1}$. By Lemma 1, (29) implies that $\Theta < 0$. With (27), we have $\mathcal{L}V(x(t), t) < 0$ for all $x(t)' \neq 0$. According to Definition 1, Definition 2 and [40], the closed loop system (6) is robustly stochastically stabilizable. □

5. Non-fragile Robust H_∞ Control

In this section, we derive a sufficient condition such that the non-fragile robust H_∞ control problem is solved.

Theorem 2. *For a prescribed $\gamma > 0$ and a symmetric matrix $N > 0$, suppose that there exist symmetric positive definite matrices $X > 0$, Y and positive scalars $\epsilon_1 > 0$, $\epsilon_2 > 0$, $\epsilon_3 > 0$, such that the LMI (30) holds, then the nonlinear stochastic system (6) with $K = YX^{-1}$ is robustly stochastically stabilizable with* *disturbance attenuation* γ,

$$\begin{bmatrix} \bar{\Psi}_1 & C & XF' & M_1 & XN_a' & BM_2 & XN_k' & T & XG' & XN_d' & XD' \\ \star & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E' \\ \star & \star & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & -\epsilon_1^{-1} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -\epsilon_2^{-1} I & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -\epsilon_2 I & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & -N & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & -X & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & -\epsilon_3 I & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \bar{\Psi}_2 \end{bmatrix} < 0, \quad (30)$$

where $\bar{\Psi}_1 = AX + BY + XA' + Y'B'$, $\bar{\Psi}_2 = \epsilon_3 M_1 M_1' - X$.

Proof. Substituting the state feedback controller (13) into system (6), we have

$$\begin{aligned} dx(t) &= [\bar{A}x(t) + M_1 U_1(t) N_a x(t) + BM_2 U_2(t) N_k x(t) + Cv(t) + \Delta f(x(t))]dt \\ &\quad + [Dx(t) + M_1 U_1(t) N_d x(t) + Ev(t)]dW_1(t) + g(x(t))dW_2(t), \end{aligned} \quad (31)$$

where $\bar{A} = A + BK$. By (30) it is obvious that the LMI (16) holds, which ensures, from Theorem 1, that our closed-loop system (6) is robustly stochastically stabilizable for all admissible uncertainties. Now, let us show that the system (6) satisfies the H_∞ performance $\|z(t)\|_2 < \gamma \|v(t)\|_2$ for all admissible uncertainties. Similarly, (19) is chosen to be the Lyapunov candidate again. For $t > 0$, we define the functional

$$J(t) \triangleq \mathbb{E}\left\{ \int_0^t [z(s)'z(s) - \gamma^2 v(s)'v(s)]ds \right\}. \quad (32)$$

According to Itô's formula, we have

$$\mathbb{E}[V(x(t), t)] = \mathbb{E}\left\{ \int_0^t \mathcal{L}V(x(s), s) \right\}, \quad (33)$$

where

$$\begin{aligned} &\mathcal{L}V(x(t),t) \\ = \;& 2x(t)'P[M_1U_1(t)N_ax(t) + BM_2U_2(t)N_kx(t) + \bar{A}x(t) + Cv(t) + \Delta f(x(t))] \\ & + [Dx(t) + M_1U_1(t)N_dx(t) + Ev(t)]'P[Dx(t) + M_1U_1(t)N_dx(t) + Ev(t)] \\ & + g(x(t))'Pg(x(t)). \end{aligned} \tag{34}$$

From (33), we are able to rewrite (32) as

$$J(t) = \mathbb{E}\left\{ \int_0^t [z(s)'z(s) - \gamma^2 v(s)'v(s) + \mathcal{L}V(x(s),s)]ds \right\} - \mathbb{E}[V(x(t),t)], \tag{35}$$

which leads to the inequality

$$J(t) \le \mathbb{E}\left\{ \int_0^t [z(s)'z(s) - \gamma^2 v(s)'v(s) + \mathcal{L}V(x(s),s)]ds \right\}. \tag{36}$$

Let $P = X^{-1}$. From (30), (18) is satisfied. Again, by Lemma 2 and (10) to (12), we have

$$J(t) \le \mathbb{E}\left\{ \int_0^t \begin{bmatrix} x(s)' & v(s)' \end{bmatrix} \Lambda \begin{bmatrix} x(s)' & v(s)' \end{bmatrix}' dx \right\}, \tag{37}$$

where

$$\begin{aligned} \Lambda \;=\; & \begin{bmatrix} \Omega & PC \\ \star & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} D' \\ E' \end{bmatrix} (P^{-1} - \epsilon_3 M_1 M_1')^{-1} \begin{bmatrix} D & E \end{bmatrix} \\ & + \epsilon_3^{-1} \begin{bmatrix} N_d' \\ 0 \end{bmatrix} \begin{bmatrix} N_d & 0 \end{bmatrix}, \end{aligned} \tag{38}$$

and

$$\begin{aligned} \Omega \;=\; & F'F + P\bar{A} + \bar{A}'P + \epsilon_1 PM_1M_1'P + \epsilon_1^{-1}N_a'N_a \\ & + \epsilon_2 PBM_2M_2'B'P + \epsilon_2^{-1}N_k'N_k + PTN^{-1}T'P + L'NL + G'PG. \end{aligned} \tag{39}$$

Let us pre and post multiply (30) by $diag(P, I, I, I, I, I, I, I, I, I, I)$, then we have the matrix inequality (40),

$$\begin{bmatrix} \Psi_1 & PC & F' & PM_1 & N_a' & PBM_2 & N_k' & PT & G' & N_d' & D' \\ \star & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E' \\ \star & \star & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & -\epsilon_1^{-1} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -\epsilon_2^{-1} I & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -\epsilon_2 I & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & -N & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & -P^{-1} & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & -\epsilon_3 I & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \Psi_2 \end{bmatrix} < 0, \quad (40)$$

where $\Psi_1 = P(A + BK) + (A + BK)'P$, $\Psi_2 = \epsilon_3 M_1 M_1' - P^{-1}$. By Lemma 1, (40) implies $\Gamma < 0$. With (37), we have $\|z(t)\|_2 < \gamma\|v(t)\|_2$. According to Definition 3, the nonlinear stochastic system (6) with $K = YX^{-1}$ is robustly stochastically stabilizable with disturbance attenuation γ. □

Remark 1. *The main results, i.e. Theorem 1 and Theorem 2, are presented in the most general forms that can be applied to many other situations, such as flight control, bond pricing, etc.*

6. Numerical Simulations

In this section we provide some simulation examples to illustrate the usefulness of the theorems obtained in this paper.

From practical experience, we assume that the time constants of system (2) are as follows: (time unit second is omitted) $T_{WTG} = 1.5$, $T_{PV} = 1.8$, $T_{ER} = 1.15$, $T_{MT} = 2.5$, $T_{BES} = 0.12$, $T_{FES} = 0.1$, $S_{WTG} = 3$, $S_{PV} = 2$, $\alpha = 0.02$, $\beta = 0.2$. Then for system (6), matrices A, B, C, D, E are obtained.

We assume the remaining system coefficients of (6) as follows.

$$M_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad M_2 = 0.8, \quad N_k = \begin{bmatrix} 0.3 \\ 0 \\ 0.1 \\ 0.3 \\ 0.1 \\ 0 \\ 0.3 \end{bmatrix}',$$

$$N_a = \begin{bmatrix} 0 & 0 & 0 & 0 & -3.333 & 0 & 3.333 \\ 0 & 0 & 0 & 0 & 0 & -4 & 4 \\ -2 & -2 & -2 & -2 & -2 & -2 & -20 \\ 0 & 0 & -0.348 & 0 & 0 & 0 & 0 \\ -0.267 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.22 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.16 & 0 & 0 & 0 \end{bmatrix},$$

$$N_d = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.133 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$T = 0.2\mathbf{I}_7, \quad L = 0.1\mathbf{I}_7, \quad G = 0.3\mathbf{I}_7, \quad N = \mathbf{I}_7,$$

First, our purpose is to design a controller such that the system of DC MG is stable when all the potential parameter uncertainties are taken into account. Mathematically, for the nonlinear stochastic robust stabilization problem in Section 4, our purpose is to design a non-fragile state feedback controller such that the controlled system is robustly stochastically stable for the admissible uncertainties $\Delta A(t)$ and $\Delta D(t)$. The Matlab LMI Control Toolbox is used to solve the LMI (16). The results are obtained as follows,

$$X = \begin{bmatrix} 5.7276 & -0.2634 & -0.1536 & -0.2404 & -0.3191 & -0.3307 & -0.3908 \\ -0.2634 & 4.5786 & -0.0706 & -0.0420 & -0.3146 & -0.3230 & -0.3260 \\ -0.1536 & -0.0706 & 6.4508 & -0.3603 & -0.2584 & -0.2741 & -0.4492 \\ -0.2404 & -0.0420 & -0.3603 & 6.5866 & -0.3278 & -0.3408 & -0.4628 \\ -0.3191 & -0.3146 & -0.2584 & -0.3278 & 1.6492 & 0.0897 & 0.1264 \\ -0.3307 & -0.3230 & -0.2741 & -0.3408 & 0.0897 & 1.4721 & 0.1483 \\ -0.3908 & -0.3260 & -0.4492 & -0.4628 & 0.1264 & 0.1483 & 0.7306 \end{bmatrix},$$

$$Y = \begin{bmatrix} -0.7032 & -0.1563 & 1.3520 & -2.4060 & 0.2520 & 0.3625 & 2.4656 \end{bmatrix},$$

$\epsilon_1 = 7.2374$, $\epsilon_2 = 7.4028$, $\epsilon_3 = 0.4864$. According to Theorem 1, the non-fragile robust stabilization problem is solvable, and the matrix in control (13) is

$$K = \begin{bmatrix} 0.1622 & 0.2567 & 0.4806 & -0.0610 & 0.0015 & 0.0288 & 3.8266 \end{bmatrix},$$

When we apply the robust H_∞ control method to the field of EI, we would like to design a controller such that voltage control problem is accomplished subject to external disturbances from wind, solar radiation and energy routing fluctuation. Mathematically, apart from the requirement of robust stability, a prescribed H_∞ performance is required to be achieved. Using the same data illustrated above and assuming the disturbance attenuation $\gamma = 0.9$, we solve the LMI (30) as follows,

$$X = \begin{bmatrix} 183.4946 & -37.2029 & -44.6694 & -43.5199 & -5.1345 & -4.8872 & -2.5765 \\ -37.2029 & 124.8440 & -32.0690 & -28.5244 & -3.2578 & -3.1816 & -1.1194 \\ -44.6694 & -32.0690 & 249.1533 & -32.3942 & -3.8541 & -3.3047 & -5.1339 \\ -43.5199 & -28.5244 & -32.3942 & 208.1204 & -4.9923 & -4.4284 & -3.9003 \\ -5.1345 & -3.2578 & -3.8541 & -4.9923 & 54.4424 & 1.4888 & 1.4938 \\ -4.8872 & -3.1816 & -3.3047 & -4.4284 & 1.4888 & 49.2576 & 1.7354 \\ -2.5765 & -1.1194 & -5.1339 & -3.9003 & 1.4938 & 1.7354 & 22.8419 \end{bmatrix},$$

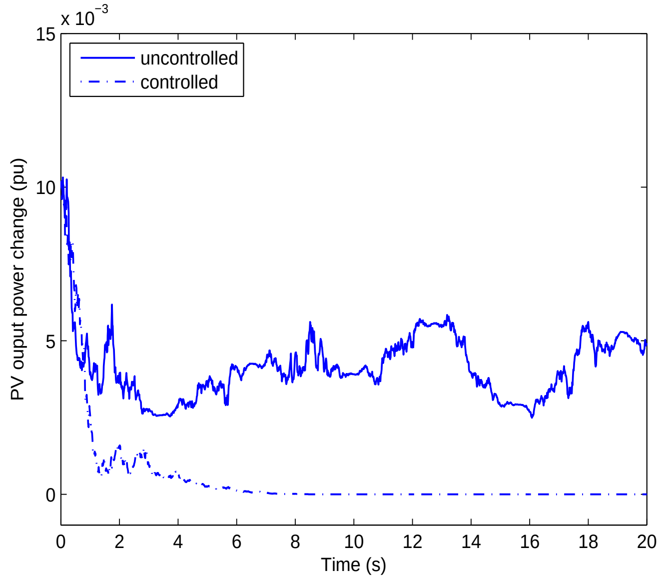

Figure 2: the Uncontrolled and Controlled Output Power Change of PV

$$Y = \begin{bmatrix} -110.323 & -38.086 & -5.986 & -179.593 & -64.498 & -53.081 & 388.615 \end{bmatrix},$$

$\epsilon_1 = 282.8136$, $\epsilon_2 = 261.6877$, $\epsilon_3 = 19.2331$. Therefore, according to Theorem 2, we can see that the non-fragile robust H_∞ control problem is solvable, and the matrix in control (13) is

$$K = \begin{bmatrix} -0.8542 & -0.7370 & -0.0950 & -0.9222 & -1.8157 & -1.8419 & 16.9607 \end{bmatrix}.$$

Next, we use Matlab to plot the graphs of the uncontrolled as well as the controlled output power change of PV, WTG, MT, BES, ER and FES in Fig. 2, Fig. 3, Fig. 4, Fig. 5, Fig. 6 and Fig. 7, respectively. The comparison results of the uncontrolled and controlled DC bus voltage deviation is plotted in Fig.8. The simulation results show that the controlled output power change of these PV, WTG, MT, BES, ER and FES as well as the DC bus voltage deviation satisfies the robust stability under a prescribed H_∞ performance, indicating the effectiveness and flexibility of the proposed method.

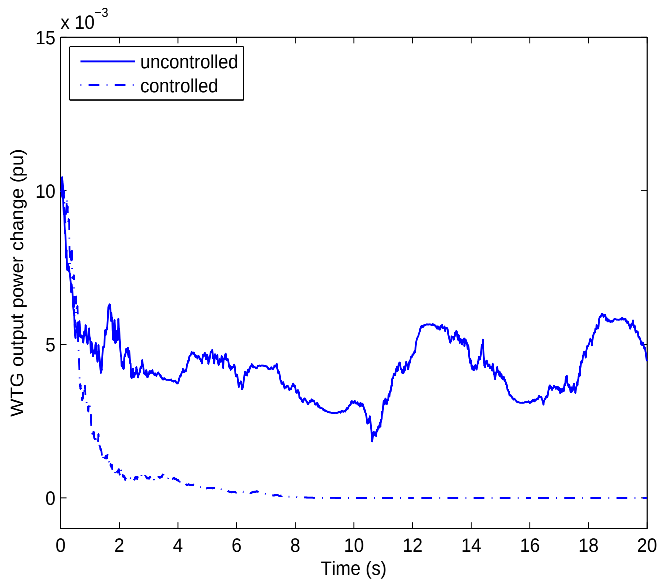

Figure 3: the Uncontrolled and Controlled Output Power Change of WTG

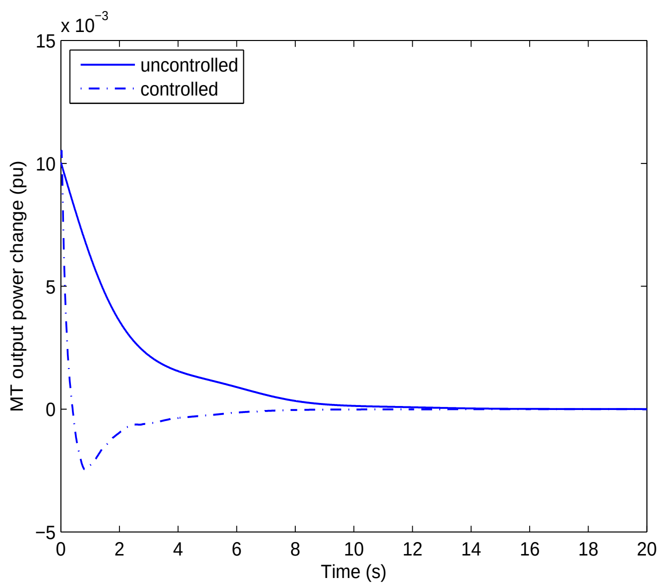

Figure 4: the Uncontrolled and Controlled Output Power Change of MT

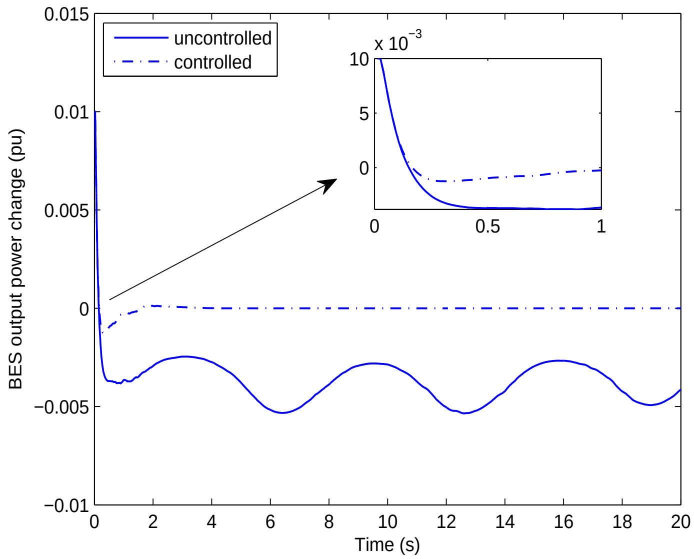

Figure 5: the Uncontrolled and Controlled Output Power Change of BES

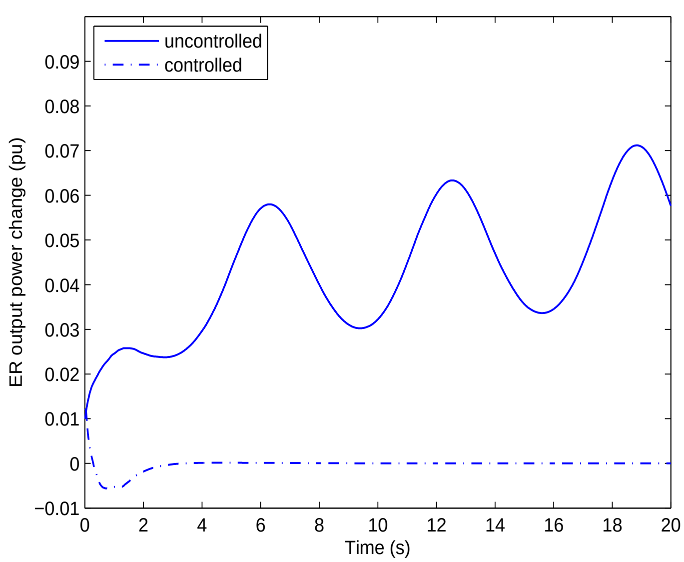

Figure 6: the Uncontrolled and Controlled Output Power Change of ER

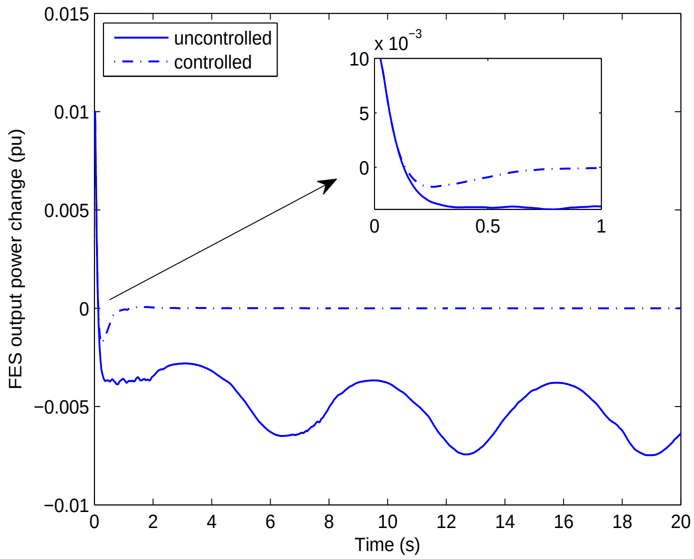

Figure 7: the Uncontrolled and Controlled Output Power Change of FES

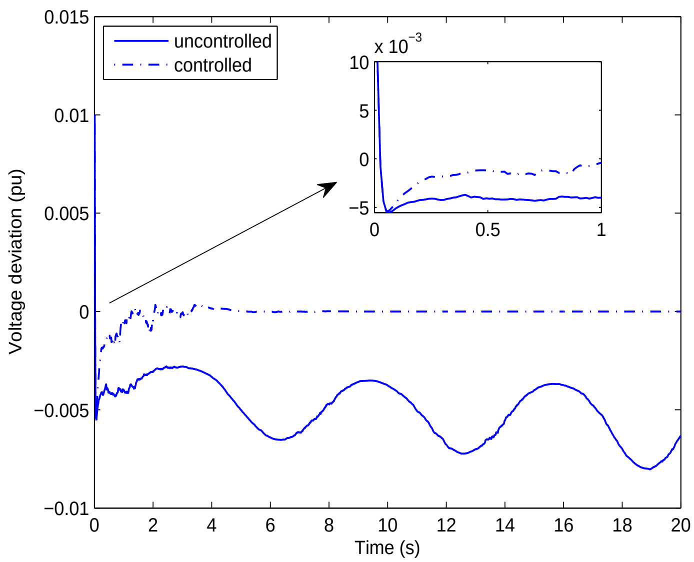

Figure 8: the Uncontrolled and Controlled DC Bus Voltage Deviation

7. Conclusion

This paper investigates the voltage control problem for the off-grid DC MG system within the scenario of EI. We propose a group of differential equations to model such DC MG system, where ODEs are used to model the output power change of ER, MT, BES and FES, and SDEs are used to model the output power change of WTG, PV and load. We combine the above differential equations into one single nonlinear SDE. Specifically, the nonlinearities appear in both drift and diffusion parts of such SDE. We allow parameter uncertainty in the integrated system of DC MG, and the controller is set to be non-fragile. The voltage control problems of the DC MG are transformed into the problems of non-fragile robust stochastic stabilization and non-fragile robust H_∞ control, which are solved via LMI approach. Numerical examples are illustrated, and simulation results show that our aim is achieved.

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